Fryderyk Mirota
Natalia Nehrebecka

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ArchaeGraph
Wydawnictwo Naukowe

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AUTHORS

FRYDERYK MIROTA
NATALIA NEHREBECKA, FACULTY OF ECONOMIC SCIENCES, UNIVERSITY OF WARSAW
NNEHREBECKA@WNE.UW.EDU.PL, ORCID: 0000-0003-3870-7231

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INTRODUCTION

Modern economic research increasingly uses modern analytical methods. A special class of exploited econometric models are dynamic models estimated on panel data[1]. They are characterised by a significant effect on the dependent variable of the time factor, which is taken into account by using lagged values of selected variables as regressors (most often an explanatory variable lagged by one period).

These models play a key role in corporate finance research. They make it possible to take into account the dynamic nature of many of the economic phenomena in the subject under consideration. They are most often used to study the capital structure of companies and the size of their cash holdings, but also for dividend payment policy or investment in fixed assets. Despite such a wide range of applications of dynamic panel models in relation to the topic of corporate finance, their estimation still poses many difficulties. This has to do with the very specific nature of the data used in the research. Firstly, it is possible to observe in them an important heterogeneity of firms, which is reflected by the individual effect included in the estimated model (independent of time). More importantly, however, endogeneity of some explanatory variables is common in corporate finance research. Furthermore, due to the existence of correlations between the individual effect of the subjects and the lagged dependent variable introduced into the modelling, the standard estimation methods used for static models (pooled OLS estimator, fixed effects estimator and random effects estimator) will not be consistent. This rules out their applicability to dynamic modelling.

[1] In this paper, these models are also referred to as dynamic panel models.

The purpose of this monograph is to present the development of estimation methodologies for dynamic models estimated on panel data, to compare their properties in relation to corporate finance research, and to provide practical guidance for authors of empirical articles to improve the estimation quality of the models they consider.

A line of research emphasising comparisons of estimator properties for dynamic panel models in relation to corporate finance issues was pioneered by Flannery and Hankins[2], who pioneered the use of *Monte Carlo* simulations to examine the properties of estimators for dynamic panel models in relation to the topic of optimal capital structure of US firms. Subsequently, Zhou et al.[3] and Dnag et al.[4] extended this approach by comparing two groups of estimators: standard ones dedicated to dynamic panel models and their counterparts enriched by the application of biased correction. Elsas and Florysiak[5], in turn, drew attention to the fact of the two-sided limitation of the analysed firm characteristics. They suggested an estimation method addressing this problem and compared the properties of the newly proposed estimator with previous approaches. Let us note that, despite the interest of researchers in the issues discussed in relation to corporate finance, this is a relatively new field and there are still issues within which a large number of aspects remain incompletely explored.

The authors of the above-mentioned studies considered the properties of estimators for dynamic panel models in the context of the properties of parameter estimators with a lagged dependent variable. The present study also takes this approach. The focus on the aforementioned parameter is particularly important, as it is its biased that can mainly determine the economic conclusions of the study. This is because it is responsible for the fundamental difference between the two leading economic theories relevant to the topic under consideration

[2] M. J. Flannery, K. W. Hankins, *Estimating Dynamic Panel Models in Corporate Finance*, Journal of *Corporate* Finance, 2013, vol. 19, pp. 1-19.

[3] Q. Zhou, R. Faff, K. Alpert, *Bias Correction in the Estimation of Dynamic Panel Models in Corporate Finance*, Journal of *Corporate* Finance, 2014, vol. 25, pp. 494-513.

[4] V. A. Dnag, M. Kim, Y. Shin, *In Search of Robust Methods for Dynamic Panel Data Models in Empirical Corporate Finance*, Journal of Banking and Finance, 2015, Vol. 53, pp. 84-98.

[5] E. Elsas, D. Florysiak, *Dynamic Capital Structure Adjustment and the Impact of Fractional Dependent Variables*, Journal of Financial and Quantitative Analysis, 2015.

(the substitution theory and the hierarchy of funding sources theory). Consequently, the selection of an adequate estimation method for a dynamic panel model can be crucial for economic research in corporate finance.

Therefore, the **main hypothesis** of this monography is that, despite continuous improvements in methodologies for estimating dynamic models on panel data, it is not possible to unambiguously identify the best estimation method for empirical studies in corporate finance based on this type of model. However, it is possible to identify a rationale that allows, in some cases, to indicate the most appropriate estimation method for the issue under consideration (*Hypothesis MH*[6]).

In order to facilitate the process of verifying the main hypothesis, three auxiliary hypotheses were put forward, relating to the basic characteristics of the dynamic model and the data used, which may affect the considered properties of the estimators. Namely, it was decided to verify the hypothesis that the lack of variation in the strength of the effect of individual explanatory variables on the dependent variable may result in a reduction in the biased and improvement in the precision of estimates of the parameter standing with the lagged dependent variable (*Hypothesis H1*). In addition, the statement that the length of the panel adopted for the study determines the choice of an adequate estimation method was tested (*Hypothesis H2*). The last auxiliary hypothesis postulates that the presence of a correlation between the subject's individual effect and the initial values of the explanatory variable significantly narrows the spectrum of possible estimation methods for dynamic models on panel data (*Hypothesis H3*).

The first chapter of the monograph emphasises issues related to how dynamic panel models are used in corporate finance research. It presents the motivations for the use of the aforementioned models, embedded in economic theory regarding the capital structure of companies and their cash holdings. In addition, the advantages and problems associated with estimating dynamic models on panel data are indicated. In the following section, examples of empirical studies in the field of corporate finance using estimators dedicated to dynamic panel models are discussed. In addition, literature items comparing the properties

[6] For convenience and greater clarity in verifying the research hypotheses, it was decided to label them uniquely.

of estimators of the parameter responsible for the speed of adjustment of the studied characteristic depending on the adopted estimation method are considered and the validity of such comparisons is justified.

Chapter two provides an extensive discussion of the evolutionary path of estimation methods dedicated to dynamic panel models. It presents in detail: the Anderson and Hsiao estimator (as pioneered in this field), the Arrellano-Bond first-difference estimator, the Blundell-Bond systematic generalised method of moments estimator and the suboptimal systematic generalised method of moments estimator. The method of variance estimation using the adjusted Windmeijer variance estimator is also presented. In addition, the advantages and disadvantages of the individual methods are indicated and the basic tests from their diagnostic process (Arellano-Bond test, Sargan test and differential Sargan test) are considered.

The third chapter verifies all the research hypotheses of this paper. This was done based on an empirical study of the properties of estimators for dynamic panel models, performed on the basis of a real economic issue - the size of the cash holdings of listed companies in Poland. Using data from the financial statements of these entities, derived from the *NOTORIA Poland* database, *Monte Carlo* simulations were carried out to investigate the properties of individual estimators of the parameter with the lagged dependent variable and to propose guidelines relating to the choice of the most appropriate estimation method (depending on the characteristics of the research sample held). The simulation experiments performed examined the effect of the true magnitude of a parameter with a lagged dependent variable on the properties of its estimators. The significance of the power of the other regressors, the length of the panel held, and the distribution of the individual effect and the purely random error for the estimation of the parameter with the lagged explanatory variable were then discussed.

The paper presents a novel approach in terms of the *Monte Carlo* experiments carried out, as they are based as much as possible on real data and not, as in most empirical studies to date, on an *AR(p)* class process. This made it possible to preserve the structure of the real data. Furthermore, an innovative research sample on listed companies in Poland was used, analysing it in the context of the cash holdings of companies in this group. In previous literature items comparing the properties of estimators for dynamic panel models, the authors use only the *Compustat* database, which makes the considered group of studies airtight due to the characteristics of the sample relevant for modelling. In addition, the researchers carry out analyses mainly based on the topic of optimal capital structure of firms. The present work at least partially fills this gap.

For greater clarity of the content presented, some of the tables and figures are included in the appendix, preceding their numbering with the letters *A* and *B* respectively. In addition, the names of variables used in the empirical part of the study, which appear in the main text, are *in italics*.

CHAPTER 1

DYNAMIC PANEL MODELS IN CORPORATE FINANCE RESEARCH

This chapter focuses on the presentation of the validity and use of dynamic panel models in empirical corporate finance research. In its first part, the motivation for the use of the aforementioned models is presented in relation to economic theories concerning the cash holdings and the capital structure of the firm. Furthermore, the econometric advantages of using panel data in research are considered, and problems in the estimation procedure resulting from the inclusion of the dynamics of the analysed phenomenon in modelling are mentioned. The second part of the chapter presents empirical examples of studies of cash holdings and capital structure of a company, using dynamic models estimated on panel data for inference. In addition, literature items comparing the properties of estimators of the parameter responsible for the speed of adjustment of the studied characteristic depending on the adopted estimation method are also presented, as well as a justification for conducting such comparisons.

1.1. Dynamic panel models and corporate finance issues

This subchapter motivates the rationale for using dynamic panel models in corporate finance research. It presents the theoretical economic models that determine the necessity of introducing the dynamics of the phenomenon under consideration into the modelling. These are, in particular, the theory of substitution and the theory of the hierarchy of sources of finance. It also points out the econometric advantages of using panel data in research and considers the problems of compatibility of standard estimators for panel models, resulting from including the dynamics of the analysed phenomenon in modelling.

1.1.1. The validity of using dynamic panel models in corporate finance research

In empirical research in corporate finance, many phenomena are considered on the basis of an analysis of their dynamics. These are mainly aspects related to the manner of financing the company's broader activities. In particular, researchers deal with the size of the company's cash holdings and the issue of the company's capital structure (examples of empirical studies for these two issues are presented in *Subchapter 1.2.1.*). To a somewhat lesser extent, these models are also used for the topics of dividend payments (e.g. study by Andres et al.[7]) and fixed asset investments (e.g. study by Aivazian et al.[8]). As the most entrenched studies in the literature analysing the dynamics of the phenomenon under study are articles dealing with cash holdings and corporate capital structure, it was decided to adopt these two issues for further consideration motivating the validity of using dynamic panel models in corporate finance research. The decision to consider these two areas of corporate finance research was made in view of the largest number of empirical studies in this area and, consequently, the largest potential audience for the conclusions of this monograph. Furthermore, the other issues in corporate finance, in the study of which dynamics are introduced into the modelling, are based on very similar assumptions, so narrowing the consideration to only the two most relevant company characteristics is not abusive. In addition, this affects the clarity of the paper, which nevertheless focuses mainly on the properties of the estimators and not solely on the economic context. Furthermore, in *Chapter 3*, for the purpose of *Monte Carlo* simulations designed to assess the properties of the selected estimators, the issue of cash holdings of listed companies was also taken as a basis for analysis (which is a certain innovation in the context of the research presented in *Subchapter 1.2.2.*).

The basic idea of the modelling, taking into account the dynamics of the corporate finance phenomenon under consideration, assumes that the

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[7] Ch. Andres, A. Betzer, M. Goergenb, L. Renneboog, *Dividend policy of German firms: A panel data analysis of partial adjustment models*, Journal of Empirical Finance, 2009, Vol. 16, pp. 175-187.

[8] V. A. Aivazian, Y. Ge, J. Qiu, *Debt Maturity Structure and Firm Investment*, Financial Management, 2005, Vol. 34, pp. 107-119.

characteristic under consideration (y) adjusts to its optimal level (y^*) according to the following equation:

$$y_{it} - y_{it-1} = \lambda(y_{it}^* - y_{it-1}) + \varepsilon_{it}, \tag{1}$$

where λ is the parameter responsible for the rate of adjustment of the quantity under study over time. Its values belong to the interval $[0,1]$ and can be interpreted as the percentage of the difference between the current (in period t) level of the quantity under study and its optimal state y^*, which the firm bridges in the course of one period. Delayed adjustments are generally due to market failures (as discussed later in this subchapter). Purely random error, on the other hand, is represented by $\varepsilon_{it} \sim iid(0, \sigma_\varepsilon^2)$. Furthermore, it is further assumed that the optimal level of the analysed quantity y^* depends linearly on the characteristics of the company:

$$\mathrm{y}_{\mathrm{it}}^* = \sum_{\mathrm{k}} \gamma_k x_{kit} + \eta_i, \tag{2}$$

where x_{kit} is the value of the k^{th} explanatory variable for *the* i^{th} firm in period t, while η_i is the time invariant individual effect of *the* i^{th} firm, illustrating its heterogeneity. By now substituting the equation (eq. 2) into the equation (eq. 1) we obtain:

$$y_{it} = \rho y_{it-1} + \sum_{\mathrm{k}} \beta_k x_{kit} + c_i + \varepsilon_{it}, \tag{3}$$

where $\rho = 1 - \lambda, \beta_k = \lambda\gamma_k, c_i = \lambda\eta_i$.

The equation of the form (eq. 3) is the final equation adopted for modelling purposes in corporate finance research, taking into account the dynamics of the phenomenon under study (the inclusion of said dynamics in the equation (eq. 3) can be seen by adopting y_{it-1} as the explanatory variable for y_{it}). This model is sometimes referred to as the *partial adjustment model*. Note that the designations from the above equations, remain in force in the following chapters, unless explicitly stated otherwise.

Let us now turn to a discussion of economic theory, which mainly determines the adoption of the equation of the form (eq.3) for modelling, *i.e.* it is the motivator for introducing the dynamics of the analysed phenomenon into consideration. Starting with the issue of cash holdings, let us first define what this term formally means and where to place it in the overall issue of corporate liquidity. Namely, in view of the frequent lack of synchronisation of expenses and receipts for executed transactions, companies should maintain a so-called liquidity reserve (otherwise known as liquidity level)[9]. This is essentially the portion of a company's assets that can be mobilised without significant financial and time losses in order to carry out transactions[10]. The liquidity level includes cash and cash equivalents and other current assets. The liquidity level can therefore be decomposed into two components: the cash holdings (consisting of cash with its equivalents) and the additional liquidity reserve (consisting of other current assets, such as short-term financial market products).

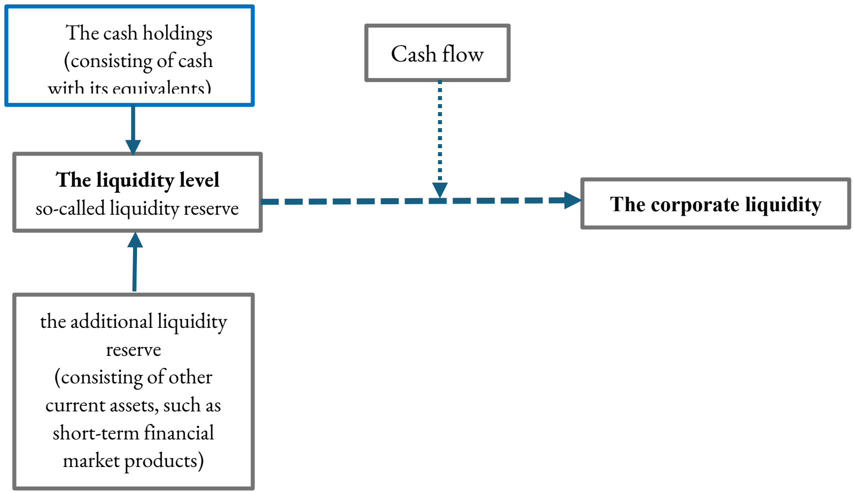
In addition, the term corporate liquidity, which is a broader concept than the level of liquidity under consideration, is directly related to the liquidity reserve. Liquidity is usually understood as "the ability of an enterprise to purchase all types of goods and services when they are needed to meet its production needs, and the ability to settle all financial obligations of the enterprise in full and within the applicable deadlines"[11]. The links between the elements of liquidity reserve and liquidity are presented graphically in *Figure 1.*

[9] G. Michalski, *Płynność finansowa w małych i średnich przedsiębiorstwach*, Warsaw: Wydawnictwo Naukowe PWN, 2013, ISBN 978-83-01-17289-3, pp. 43.

[10] ibidem, pp. 44.

[11] U. Wojciechowska, *Płynność finansowa polskich przedsiębiorstw w okresie transformacji gospodarki. Aspekty mikroekonomiczne i makroekonomiczne*, Warsaw: Oficyna Wydawnicza SGH 2001, ISBN 83-7225-098-7, pp. 14.

Figure 1. Graphic illustration of the links between liquidity reserve elements and liquidity.



Source: own study.

The motivation for companies to maintain a cash holdings at a non-zero level is market imperfections. For example, transaction costs and information asymmetries reduce the real rate of return on investment in a project. This negative effect can be offset by the company having a cash holdings. In addition, companies with sufficient cash reserves will not have to bear the costs of raising external financing. The existence of an agency problem that affects the market position of a company should also be raised here. Namely, company managers may maintain elevated levels of the most liquid assets in order to achieve their own intentions (which are at variance with the expectations of the company's owners). This problem generates additional costs for the company, for example, such as the monitoring of managers.

In relation to the discussed benefits and costs of a company maintaining a cash holdings, three main theories attempting to explain the motivation of companies to hold cash are defined in the literature.

The first of these, called substitution theory (or *trade-off* theory), specifically emphasises the existence of transaction costs, which influence firms' decisions on the volume of most liquid assets to be held. Miller and Orr[12], who authored the model in question, state that there is a certain optimal level of cash holdings that a firm should hold, and this is the level implied by the equality of marginal costs and marginal benefits of holding cash. In this sense, a firm should strive for a certain optimal level of cash holdings and therefore vary its size from period to period in order to reach the optimal state.

The potential costs that may arise from holding the most liquid assets are first and foremost a lower rate of return than if the funds in question were allocated to more profitable investments (the cost is therefore understood here in terms of opportunity cost). In addition, the high volume of cash held by the company may result in an exacerbation of the agency problem, which creates new costs for the company. The benefits of maintaining a cash holdings, on the other hand, are primarily the lower probability of the company's financial problems and the yield from not incurring borrowing costs.

In opposition to the substitution theory is the theory of the hierarchy of sources of finance, by Myers and Majluf[13], which emphasises the market failure of information asymmetry. The authors postulate that firms should use first the sources of financing for operations and investments characterised by the smallest information gap, resulting from the greater knowledge of the firm possessed by its managers than by potential investors. Accordingly, companies should first finance their activities from their own resources (which may be largely available in the form of a cash holdings), then from external sources of financing (e.g. bank loans) and, at the very end, by raising capital through share issues. What differentiates the two above-mentioned theories is that the hierarchy of funding sources theory does not postulate the existence of any optimal level at which a firm would hold cash. In this sense, between the two theories, apart from the difference in economic postulates, there is also a dispute over the validity of considering the dynamics of the phenomenon under study.

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12 M. H. Miller, D. Orr, *A Model of the Demand for Money by Firms*, Quarterly Journal of Economics, 1966, Vol. 80, pp. 413-435.

13 S. Myers, N. Majluf, *Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have*, Journal of Financial Economics 13, 1984, pp. 187-221.

The last of the popular theories attempting to explain the level of cash held by the company is the free cash flow theory of Jensen[14]. Although it is not as central to considerations regarding the validity of introducing dynamics into the modelling of the phenomenon under study as the two theories presented above, it will be briefly discussed for the sake of completeness. The aforementioned theory is based on the assumption of an agency problem between managers and owners of a company. It consists of a divergence of interests between the shareholders and the managers of the company in terms of the amount of cash to be kept and, in addition, in the situation under consideration there are no cheap ways of monitoring employees. Managers may have an incentive to maintain a cash holdings at an inflated level, relative to that which would maximise profit (expected by the owners). This is because managers want to reduce the probability of company bankruptcy at all costs, making their position more secure. In addition, cash held in higher volumes gives managers more freedom to make management decisions. This allows them to engage in projects that could potentially not be financed from external sources (e.g. due to a poor investment appraisal by a bank).

Turning now to the analysis of a company's capital structure, let us point out that there are many similar definitions of capital, but in the sense of examining capital structure, it is assumed that the characteristic under examination is the sum of funds that have been entrusted to the company by investors[15]. This may have occurred, for example, as a result of the purchase of corporate bonds, shares or the granting of credit to the company. The volume of capital can be decomposed essentially into two parts: by its cost and by its source. The first is equity, characterised by a high cost, since investors in this case are partial owners of the company and therefore bear a significant risk and expect a premium for it (usually in the form of dividends). The second part of capital is liabilities on which the company must make interest payments. This is referred to as invested capital in the

[14] M. C. Jensen, *Agency Cost of Free Cash Flow, Corporate Finance and Takeovers*, American Economic Review, 1986, vol. 76, pp. 323-339.

[15] J. Gajdka, *Theory of capital structure and their application in Polish conditions*, Łódź: Wydawnictwo Uniwersytetu Łódzkiego, 2002, ISBN: 83-7171-580-3, pp. 19.

company (or third-party capital) and mainly consists of loans and borrowings taken out by the company.[16]

The ideas of economic theories attempting to explain the capital structure of a company, are broadly in line with the ideas of the theories discussed for the cash holdings. Only the context of their applicability is variable. Accordingly, capital structure theories will be discussed only briefly.

In relation to the issue under consideration, *trade-off* theory (also referred to in this case as bankruptcy cost theory) was proposed by Modigliani and Miller[17]. It is based on balancing the marginal costs and benefits of financing a company's activities with foreign capital. In addition to the lower expected rate of return by lenders than by shareholders, an indisputable benefit of financing a company's activities with foreign capital is the phenomenon of the so-called tax shield. It consists in the fact that the interest paid on loans reduces the tax base for legal entities, consequently increasing the potential benefits of this type of financing. Costs, on the other hand, result from the expenses associated with the possible bankruptcy of a company unable to settle its debt repayments. These costs include, for example, the cost of selling assets or the cost of litigation. As the amount of debt increases, the probability of a company going bankrupt increases, and in doing so, its potential costs also increase. This also lowers the valuation of the company. As a result of this relationship, there is a certain optimum level of debt capital at which it is no longer worthwhile to increase its level and the remaining shortfall is better covered by equity. In the light of *trade-off* theory, therefore, a certain process of adjustment of the volume of external capital should be observed in order to reach its optimal level.

Myers' hierarchy theory of funding sources[18] for a firm's capital structure is fully consistent with the analogous theory discussed for transactional liquidity provision. Firms will therefore prefer financing from sources with the least information asymmetry, i.e. in sequence: financing from equity (preferably retained earnings) and then only financing from external capital in a sequence

[16] A. Duliniec, *Structure and cost of capital in an enterprise*, Warsaw: Wydawnictwo Naukowe PWN, 2001, ISBN: 978-83-01-14332-9, pp. 14.

[17] F. Modigliani, M. H. Miller, *Corporate Income Taxes and the Cost of Capital: A Correction*, American Economic Review, 1963, Vol. 53, pp. 433-443.

[18] S. C. Myers, *The Capital Structure Puzzle*, The Journal of Finance, 1983, Vol. 39, pp. 575-592.

corresponding to the increase in the information gap of the source (credit, corporate bonds, equity). The equivalent of the free cash flow theory for the subject of a company's capital structure is the agency cost theory. It too assumes a conflict of interest between managers and owners of the company. The managers of a company are inclined to avoid risky projects and prefer to finance the business from equity to a greater extent than would result from profit maximisation. This generates additional risk premium costs.

In summary, the economic theories presented on the size of firm cash and the capital structure of the firm have different conceptual bases. This implies a fundamental difference in the approach to the incidence of an optimal level of the size under consideration between the *Trade-off* theory and the hierarchy of funding sources theory. The former postulates the existence of an optimum level of the characteristic under consideration (e.g. cash holdings). As such, it should be subject to a process of adjustment over time, and thus it is reasonable to introduce dynamics into the modelling of the issue under consideration. The theory of the hierarchy of sources of funding, on the other hand, states that the optimal level of the size under consideration does not exist. In this sense, the introduction of dynamics into modelling is not theoretically necessary, but it should be noted that due to the occurrence of short-term shocks (e.g. macroeconomic) or changes in the management policy of a company, companies acting in accordance with the postulates of the theory of hierarchy of sources of financing will also have to adjust the level of the analysed size to the new realities. Ultimately, however, there is some opposition between the two theories, and empirical research (including that presented in the *Subchapter 1.2.1*) is dominated by the desire to verify which economic theory is superior in explaining the variability of the analysed phenomenon. In this context, it is also necessary to introduce dynamics modelling to verify the suggestion of *Trade-off* theory that there is an optimum level of the variable under study and that there are adjustments over time to this level.

The validity of introducing dynamics into corporate finance models, which follows directly from economic theory, makes it necessary to base modelling on panel data. This approach provides additional econometric advantages (over models based on cross-sectional samples), such as a reduction in the collinearity problem (due to the fact that the number of degrees of freedom increases), a reduction in the bias on estimators, or a more comprehensive interpretation of the model, due to the presence of information about the be haviour of individual entities in the data set. However, the greatest additional advantage, in connection with the use of panel data in the study, is the possibility to remove the time-invariant, subject-specific, unobservable individual effect from the model (in other words, firm heterogeneity is thus controlled for).[19]

In summary, the introduction of dynamics into modelling in corporate finance research allows a number of advantages to be achieved, both on the ground of verification of the postulates of economic theory and on econometric grounds. Unfortunately, it also entails significant disadvantages in the estimation procedure. In particular, standard estimators for panel data (fixed-effects and random-effects estimator, OLS estimator for panel data) should not be applied to dynamic problems. This problem will be considered in the next subchapter.

1.1.2. Econometric modelling problems

As mentioned above, standard models for static panels should not be used for dynamic models. This is due to the loss of consistency of these estimators for dynamic models, due to the correlation between the firm's individual effect and the one-period lagged dependent variable introduced into the model as an explanatory variable. This subchapter will consider the loss of fit of the aforementioned standard models used for static models.

For the fixed effects estimator and the random effects estimator, the reasoning will be carried out jointly, first based on an autoregressive model of the following form:

$$y_{it} = \rho y_{it-1} + c_i + \varepsilon_{it}. \tag{4}$$

[19] J. Lee, *Panel Data Econometrics: Methods-of-Moments and Limited Dependent Variables*, San Diego-San Francisco-New York: An Elsevier Science Imprint, 2002, ISBN: 978-0124406568.

In this case, the general form of the considered parameter estimator ρ, is:

$$\hat{\rho} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it} - \bar{y}_i)(y_{it-1} - \bar{y}_{-1})}{\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1} - \bar{y}_{-1})^2}, \tag{5}$$

where $\bar{y}_i = \frac{1}{T}\sum_{t=1}^{T} y_{it}$, while $\bar{y}_{-1} = \frac{1}{T}\sum_{t=1}^{T} y_{it-1}$.

By now substituting the equation (eq.4) into the equation (eq.5), we finally obtain:

$$\hat{\rho} = \rho + \frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(\varepsilon_{it} - \bar{\varepsilon}_i)(y_{it-1} - \bar{y}_{-1})}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(y_{it-1} - \bar{y}_{-1})^2}. \tag{6}$$

Nickell[20] showed that the numerator of a fraction in (eq. 6) converges by probability for $N \to \infty$, to some non-zero quantity. Namely:

$$\begin{aligned}\operatorname*{plim}_{N\to\infty} \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(\varepsilon_{it} - \bar{\varepsilon}_i)(y_{it-1} - \bar{y}_{-1}) \\ = -\frac{\sigma_\varepsilon^2((1-\rho)T - 1 + \rho^T)}{T^2(1-\rho)^2} < 0,\end{aligned} \tag{7}$$

where $T > 2, \rho \in [0,1]$, while σ_ε^2 represents the variance of ε_{it}.

It can be seen from the above that the estimators under consideration agree only in the case of $T \to \infty$, but such trials are not encountered in practice. Furthermore, as can be seen from the model (eq.7), the bias on the estimator is negative under the assumptions made. The considerations presented for the case

[20] S. Nickell, *Biases in Dynamic Models with Fixed Effects*, Econometrica, 1981, vol. 49, pp. 1417-1426.

of the model (eq.3) are generalised by Sevestre and Trognon[21]. The formulas they obtain become more complicated compared to those presented above, but the idea of the reasoning and the conclusions (loss of fit) remain the same. However, it additionally turns out that if the entities under study are heterogeneous and therefore the structural parameters β_k in the model (eq.3) are differentiated with respect to these entities, the fixed and random effects estimators will not agree even when $N \to \infty, T \to \infty$[22]. This fact completely invalidates the applicability of the fixed and random effects estimator for dynamic models estimated on panel data.

Turning to the analysis of the *pooled OLS* estimator in the context of dynamic modelling, let us note that, as was the case with the fixed and random effects estimator, this estimator for dynamic models loses[23] consistency due to the presence of a correlation between the individual effect and the lagged dependent variable. The estimator itself for the autoregressive model (eq. 4) takes the following form:

$$\hat{\rho}_{POLS} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} y_{it} y_{it-1}}{\sum_{i=1}^{N} \sum_{t=1}^{T} y_{it-1}^2}. \tag{8}$$

Based on the above equally (eq.4), we get:

$$\hat{\rho}_{POLS} = \rho + \frac{\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (c_i + \varepsilon_{it}) y_{it-1}}{\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it-1}^2}. \tag{9}$$

The asymptotic biased of the estimator under consideration is therefore equal to the limit by probability of the second component of the sum in the

[21] P. Sevestre, *A*. Trognon, *A Note on Autoregressive Error Components Models,* Journal of Econometrics, 1985, Vol. 28, pp. 231-245.

[22] M. H. Pesaran, R. P. Smith, *Estimating Long-run Relationships from Dynamic Heterogeneous Panels*, Journal of Econometrics, 1995, Vol. 68, pp. 79-113.

[23] For static models, this estimator is admittedly inefficient; however, it is characterised by consistency.

formula (eq. 9). Hsiao[24], as a result of his analysis, shows that the numerator and denominator of the aforementioned component of the sum, converge by probability to the following quantities:

$$\underset{N\to\infty}{\text{plim}}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(c_i+\varepsilon_{it})y_{it-1}=\frac{1-\rho^T}{T(1-\rho)}cov(y_{i0},c_i)+ \\ +\frac{\sigma_c^2}{T(1-\rho)^2}((1-\rho)T-1+\rho^T), \tag{10}$$

$$\underset{N\to\infty}{\text{plim}}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it-1}^2= \\ =\frac{1-\rho^{2T}}{T(1-\rho^2)}\sum_{t=1}^{T}\frac{y_{i0}^2}{N}+\frac{\sigma_c^2}{T(1-\rho)^2}\left(T-\frac{2(1-\rho^T)}{1-\rho}+\frac{1-\rho^{2T}}{1-\rho^2}\right)+ \\ +\frac{2}{T(1-\rho)}\left(\frac{1-\rho^T}{1-\rho}-\frac{1-\rho^{2T}}{1-\rho^2}\right)cov(y_{i0},c_i)+ \\ +\frac{\sigma_c^2}{T(1-\rho^2)^2}((1-\rho)T-1+\rho^T), \tag{11}$$

where σ_c^2 and σ_ε^2 represent the variance of the individual effect and the purely random component, respectively, while y_{i0} are some predetermined initial values. Consequently, $cov(y_{i0},c_i)>0$, and consequently the value of the estimator $\hat{\rho}_{POLS}$ will be biased upwards, with the biased being higher the higher the variance of the individual effect σ_c^2. The above conclusions are also true for smaller samples, as shown using *Monte Carlo* simulations, by Nerlove[25]. These results are also correct when the autoregressive model is extended additional explanatory variables, to a model of the form (eq. 3).

The above-described loss of consistency of standard estimators for panel data, when used for dynamic considerations, has determined the emergence

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[24] C. Hsiao, *Analysis of Panel Data*, New York: Cambridge University Press, 2003, ISBN 978-0-521-52271-7, pp. 73-74.

[25] M. Nerlove, *Experimental Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross-Sections*, Economic Studies Quarterly, 1967, Vol. 18, pp. 42-74.

of a new class of models that will be consistent, despite the introduction of a lagged dependent variable into the set of explanatory variables. The new estimation methods are mostly based on the generalised method of moments and the instrumental variables method. A very detailed description of the estimation methodology for dynamic panel models, together with a consideration of the advantages and disadvantages of the different estimators, is presented in *Chapter 2*.

Let us further note that the introduction of a dependent variable lagged by one period into the set of regressors is responsible for the loss of fit of the underlying models for panel data in the dynamic case. Therefore, the estimator of the parameter ρ standing by this variable, is most interesting in the context of considering the properties of estimators for dynamic panel models. Moreover, from the point of view of economic research, this coefficient represents the main difference between *trade-off* theory and source theory, which is the assumption of the existence (or non-existence) of an optimal level of the size under consideration, to which the firms under consideration aspire. In this sense, its estimator is also particularly important in the modelling process. In addition, the parameter $\lambda = 1 - \rho$ tells about the speed of adjustments of the size of the characteristic under study which, as Huang and Ritter state[26], is the most important aspect in today's consideration of the capital structure of companies. For the above reasons, it was decided that the main focus of this paper would be the properties of the estimators of the parameter ρ (eq.3)), and therefore the properties of the parameter estimators β_k, which are of lesser importance for the whole modelling in the econometric sense, were not considered in the deliberations. Sometimes, for the sake of simplicity, when the paper refers to the properties of the estimators of dynamic panel models, we mean the different estimators of the parameter ρ. In addition, when the parameter ρ is mentioned, each time (regardless of the chapter of the paper) it is meant as the parameter ρ in the model designations (eq.3).

Let us note that corporate finance issues, for the modelling of which the equation(3), is used, are a very specific group of problems due to several features that can significantly affect econometric modelling. The first is the already widely

[26] R. Huang, J. R. Ritter, Testing Theories of Capital Structure and Estimating the Speed of Adjustment, Journal of Financial and Quantitative Analysis, 2009, Vol. 44, pp. 237-271.

discussed use of the lagged dependent variable in modelling. In addition, companies tend to be characterised by significant heterogeneity, which is reflected in the individual effect c_i. Most significantly, however, the problem of endogeneity is common in corporate finance research. It is usually caused by the possibility of some unobservable shocks affecting both the explanatory variable and any of the explanatory variables.

In view of the characteristics of corporate finance research outlined above, it seems reasonable to consider the properties of estimators of dynamic panel models for these issues, especially in the context of the multiplicity of models for this purpose, which are characterised by different properties (see *Chapter 2*). Bias and efficiency are mentioned as particularly important properties of estimators (in addition to the compatibility discussed above). Bias is the property of equality of the expected value of the distribution of the estimator and the estimated parameter. Its occurrence can, in the worst case, lead to a regressor being considered as a significant variable, which really is not. In addition, another of its effects may be to obtain a false magnitude of the effect of the explanatory variable on the dependent variable (both effects are highly undesirable in empirical studies). The efficiency of an estimator, on the other hand, tells you the size of its variance, and a high value of this can also result in unreliable results in an empirical study. The ideal situation is to use an unconstrained estimator with the smallest possible variance. However, Ziliak[27] points out that in the case of dynamic models estimated on panel data there is some substitutability between biased and estimation efficiency (these considerations were carried out for methods based on the generalised method of moments. They indicated that, starting from a certain number of instruments, as the number of instruments increases, the efficiency of the estimator increases, but so does its bias). In the context of the above considerations, it is very important to study the properties of the estimators of the parameter ρ and to try to identify conditions that can improve the quality of estimation of the considered empirical models in corporate finance. These considerations were performed in *Chapter 3* by means of *Monte Carlo*

[27] J. P. Ziliak, *Efficient Estimation with Panel Data when Instruments are Predetermined: An Empirical Comparison of Moment-Condition Estimators*, Journal of Business and Economic Statistics, 1997, Vol. 15, pp. 419-431.

simulations, using the example of the transactional reserve of listed companies in Poland.

In summary, this subchapter presents the reasons for the loss of consistency of the fixed effects, random effects and *pooled OLS* estimators when used to estimate dynamic models on panel data. In addition, the specific characteristics of the datasets used for corporate finance research are discussed, showing that it is legitimate and necessary to consider the biased and effectiveness of dynamic model estimators used in research on this topic. In the next subchapter, sample empirical articles in the field of capital structure and cash holdings of companies will be presented, the results of which also indicate the indispensability of studying the properties of parameter estimators with a lagged dependent variable in a model of the form (eq. 3).

1.2. Examples of the use of dynamic panel models in relation to corporate finance issues

The following chapter focuses on the presentation of empirical research related to the issue at hand. It is divided into two parts. The first part discusses examples of research on cash holdings and capital structure of enterprises, in which dynamic panel models were used. The second part of the subchapter presents attempts to compare the properties of the parameter estimators ρ for different estimation methods in relation to corporate finance research.

1.2.1. Empirical articles on cash holdings and corporate capital structure

The following are examples of empirical studies in the field of cash holdings and corporate capital structure, which use dynamic panel models for modelling. Let us note that there is a very wide variety of considerations in the literature in this area, and this subchapter is only a summary presentation of a few papers in order to highlight the validity of considering the properties of the estimators used in this context. They have been selected in terms of their key relevance to the issue under consideration (pioneering articles) and their topicality.

The selection of current articles was guided by the fact that the authors use econometric models (presented in *Chapter 2*) dedicated to dynamic considerations.

Starting with analyses of transactional liquidity provision, the study by Ozkan and Ozkan[28] is considered the work that initiated the use of dynamic panel models to analyse this topic. The authors mainly focus on the impact of management structure on the amount of cash held by a firm. UK listed companies were taken as the subjects of consideration. The data used are from 1984 to 1999, and are sourced from *Datastream* (for information on the financial characteristics of companies) and *Price Waterhouse Cooper* reports (for information on company ownership and management structure). The authors, as pioneers in this field, used the Arellano-Bond first-difference estimator for modelling (in addition to linear regression on an averaged set of variables, which is not adequate for the problem under consideration). This made it possible to include dynamics in the model (for which the dependent variable lagged by one period is responsible) and to control in an econometrically correct way the problem of endogeneity, which, as Ozkan and Ozkan point out[29], arises from the simultaneity of variables. The authors adopt two extreme scenarios in their analysis, where in the first scenario they treat only the one-period lagged dependent variable as endogenous, while in the second scenario the nature of all explanatory variables is identified as endogenous. Thus, in both cases the researchers obtain significantly different parameter estimates with a lagged dependent variable. The main conclusion of the paper under review is the suggestion that the size of the company's shares held by its managers is a significant determinant of the volume of the most liquid assets held by the company. This relationship is furthermore non-monotonic in nature. In addition, the nature of the main shareholder is important for the volume of cash held by the company - on average, family-owned firms maintain a higher cash holdings than companies with institutional owners.

[28] A. Ozkan, N. Ozkan, *Corporate Cash Holdings: An Empirical Investigation of UK Companies*, Journal of Banking and Finance, 2004, Vol. 28, pp. 2103-2134.
[29] ibid.

Garcia-Teruel and Martinez-Solano[30] focus on identifying the determinants of the size of the cash reserves of small and medium-sized Spanish firms. The vast majority of previous empirical studies on transactional liquidity provision have been based on data on companies listed on regulated markets. In this sense, the study presented here is innovative in that it refers to companies in the SME sector. In this group, market imperfections, particularly information asymmetry, become much more important. These companies are more exposed to liquidity problems and are at a higher risk of bankruptcy. A model of the form(3) was estimated based on data from the financial statements of Spanish industrial companies in the SME sector from the *System of Analysis Spanish Balance Sheets* database. A two-step Arellano-Bond first-difference estimator was used (discussed in detail *in Subchapter 2.1.2.*), varying the set of independent variables slightly across models. The result of the analysis is the conclusion that the companies under consideration are moving towards the target cash level and that the speed of adjustment is significantly higher than in earlier studies (the lower value of the parameter estimate with the lagged dependent variable corresponds to a higher parameter $\lambda = 1 - \rho$ reflecting the speed of adjustment of the studied characteristic). This is related to the greater impact of market imperfections on smaller firms, resulting in higher costs of raising and servicing external finance.

The consideration of Pakistani companies in relation to the topic of cash holdings was addressed by Shah[31]. He paid particular attention to the higher level of short-term liabilities of firms in developing countries. In this context, the research questions raised were whether firms in developing countries maintain higher cash reserves (as they have higher short-term liabilities) and whether they match the maturity of their accounts payable and accounts receivable. The author bases the study on panel data from 1996 to 2008, on 280 non-financial listed companies, listed on the *Karachi Stock Exchange* (the data is from Bank of Pakistan publications). Two types of approaches, static and dynamic, were used for modelling. For the former, estimates of the fixed and random effects

[30] P. Garcia-Teruel, P. Martinez-Solano, *On the Determinants of SME Cash Holdings: Evidence from Spain*, Journal of Business Finance and Accounting, 2008, Vol. 35, pp. 127-149.

[31] A. Shah, *The Corporate Cash Holdings: Determinants and Implications*, African Journal of Business Management, 2011, Vol. 5 (34), pp. 12939-12950.

estimator and the *pooled OLS* estimator are presented. Here, a lagged dependent variable is not introduced into the model, so the estimators mentioned do not lose their consistency. Dynamic models, on the other hand, were estimated using the two-step systematic estimator of the generalised Blundell-Bond method of moments (described in *subchapter 2.1.3.*). This allowed the econometrically correct introduction of the lagged dependent variable into the modelling and controlled for the endogeneity of some regressors. In this sense, the utility of the first group of models considered is marginal. However, it should be noted that all the estimation methods indicated give ideologically similar conclusions for the independent variables, while numerous non-significant variables are identified among them, which can significantly distort the results. Ultimately, however, the researcher indicates that there is no basis for the claim that developing countries maintain higher balances of the most liquid assets. This is, among other reasons, because firms match the maturity of their receivables and payables adequately, which is some equivalent of keeping the cash holdings at a high level.

Based on the studies presented above, there is no basis for rejecting the hypothesis of the existence of an optimal level of cash holdings pursued by the entities analysed. The work of Bigelli and Sanchez-Vidal[32] even draws extreme conclusions in this respect. The authors considered private Italian non-financial companies, based on the *Italian Company Information and Business Intelligence* database (provided by *Bureau Van Dijk*). Data from 1996-2005 were used for consideration and information on financial and insurance companies was excluded. Econometric modelling was performed using a two-step Arellano-Bond first-difference estimator (presented in *Subchapter 2.1.2.*). The results obtained from the study do not clearly indicate either the superiority of the postulates of the *Trade-off* theory or the postulates of the hierarchy of funding sources theory. Nevertheless, the parameter with the lagged dependent variable used as a regressor was found to be statistically indistinguishable from zero. Consequently, $\hat{\lambda} = 1 - \hat{\rho} = 1$ and it is possible to speak, in the sense of the model (eq.3) of instantaneous adjustments. In light of the financial sector constraints to which

[32] M. Bigelli, J. Sanchez-Vidal, *Cash holdings in private firms*, Journal of Banking and Finance 36, 2012, pp. 28.

private SME firms are subject, such a pattern can be expected to be particularly pronounced for them.

Turning to the analysis of corporate capital structure articles, let us consider the Ozkan study[33], which pioneered the use of panel data to consider capital structure. The author used information on 390 UK listed companies, drawn from the *Compustat* database and covering the period 1984-1996. The aim of the study is to identify the determinants affecting a company's propensity to finance its operations with debt capital. A two-step Arellano-Bond first-difference estimator was used for the estimation, allowing for the econometrically correct introduction of a lagged dependent variable into the modelling and examining the centrality of the adjustment process to the phenomenon under study. The researcher's result indicates the existence of a certain optimal level of the external debt ratio pursued by the companies under consideration. Furthermore, the adjustment process is relatively fast (compared to previous studies of the cash holdings) and the parameter estimate with the lagged dependent variable is 0.431. This may indicate the high cost of having a suboptimal capital structure for the firm.

Xu[34], on the other hand, points out that the overwhelming majority of studies assume that the process of adjusting the capital structure to the optimal level of external capital is symmetric. In other words, for both companies increasing external financing and decreasing it, adjustments should take place at the same rate. However, such a statement is not necessarily true, hence the researcher decided to divide the sample into two subsets: firms that increase their level of external capital and firms that decrease it. Using data from the *Compustat* database, covering the period 1970-2004 and all types of firms excluding financial, insurance and agricultural activities and firms with total assets of less than $10 million, econometric models were estimated using Arellano-Bond and Blundell-Bond methods (a fixed effects model and *pooled OLS* were also used, but will not be commented on due to the inadequacy of these methods for

[33] A. Ozkan, *Determinants of Capital Structure and Adjustment to Long Run Target: Evidence from UK Company Panel Data*, Journal of Business Finance and Accounting, 2001, Vol. 28, pp. 175-198.

[34] Z. Xu, *Do Firms Adjust Toward a Target Leverage Level?,* Bank of Canada Working Paper Series, 2007, On line, http://www.bankofcanada.ca/wp-content/uploads/2010/02/wp07-50.pdf.

dynamic panel models). The conclusions obtained by the author indicate a slower process of capital structure adjustment for firms reducing the amount of external financing than for firms increasing it. As indicated in the study, this situation is not due to different optimal levels of external debt for these companies, nor to different adjustment costs or differences in the type of business. Xu points to the timing discrepancy between the ability to undertake profitable investments and the time to receive external financing as the reason for this. Hence, sometimes companies keep their debt ratio inflated. From an econometric point of view, it is particularly noteworthy that there is a large difference in the parameter estimates with the lagged dependent variable for the Arellano-Bond and Blundell-Bond estimator This may indicate a significant problem of weak instruments and a downward biased of the Arellano-Bond estimator (this issue is considered in *Subchapter 2.1.2*). Therefore, the results obtained would require additional verification and need to be treated with some caution, which the author does not mention.

A study of the capital structure of firms in comparative terms across countries was conducted by Öztekin and Flannery[35]. They considered a partial adjustment model for 37 countries. Using the *Compustat* database, information from 1991-2006 was selected, based on which models were estimated using two methods: the two-step systematic estimator of the generalised method of moments Blundel-Bond (discussed in *Subchapter 2.1.3.*) and *the Least Square Dummy Variable Corrected* Estimator (discussed in *Subchapter 2.1.6.*). In addition, for the sake of completeness of the argument, the authors adopt two explanatory variables for the analysis: the accounting leverage ratio (quotient of total liabilities and total assets) and the market leverage ratio (quotient of total liabilities and total assets less the difference between the market and book value of the company's shares). The study concludes that the differences in the speed of adjustment of the capital structure of companies from different countries, are due to different laws, different levels of development of institutions (mainly financial) and policy approaches (e.g. tax) towards companies. The authors draw attention to the length of *the half-life* of the size under consideration to the optimal level. This is the time

[35] Ö. Öztekin, M. J. Flannery, *Institutional Determinants of Capital Structure Adjustment Speeds*, Journal of Financial Economics, 2012, Vol. 103, pp. 88-112.

it takes for a company to close half of the difference between the current and optimum level of the analysed characteristic, after a unit shock in the purely random componentε_{it} of the model (eq. 3). It is determined based on a formula of the following form:

$$half - life = \frac{\ln(0,5)}{\ln(1-\lambda)} = \frac{\ln(0,5)}{\ln(\rho)}. \tag{12}$$

In the case of the countries considered, the authors identify very wide variations in this respect, ranging from a half-life of about one year to 15 years, depending on the country considered. This illustrates how important the environment in which companies operate can be for their capital structure.

The smaller variety of countries analysed in the study (only G7 countries) was limited to Drobetz et al.[36]. Again, the *Compustat* database was used in the considerations, downloading information from 1992 to 2009 on companies from the aforementioned group of countries. The authors aim to consider the difference in the speed of adjustment of the capital structure of companies in economies where the raising of foreign capital is based on the banking system and economies where external financing comes from market sources (mainly equity issuance). Various estimation methods dedicated to dynamic panel models, described in detail in *Chapter 2*, namely the Arellano-Bond estimator, the Blundell-Bond estimator, the *Long-difference Instrumental Variables Estimator*, *the Dynamic Panel Fractional Estimator* and *the Least Square Dummy Variable Corrected Estimator*, were used for modelling. Quite diverse values of the parameter estimates were obtained ρ (ranging from 0.439 to 1.062), with a worrying value for the Blundell-Bond estimator, equal to 1.062. This is uninterpretable in light of the theoretical underpinnings of the partial adjustment model and differs significantly from the values of the estimates of the other models. This may be related to the failure to meet all the assumptions for the Blundell-Bond model

[36] W. Drobetz, D. C. Schilling, H. Schröder, *International Evidence on the Heterogeneity of Capital Structure Adjustment Speeds*, European Financial Management Association, 2013, On line, http://efmaefm.org/0EFMAMEETINGS/EFMA%20ANNUAL%20MEETINGS/2013-Reading/phd/EFMA %20Doctoral%20 Seminar%202013.pdf.

(although the authors do not mention this), as discussed in more detail in *Subchapter 3.3.4*. Ultimately, however, the researchers conclude that capital raising is slower for countries where it is mainly done through the banking system than for countries where firms raise external finance from market sources. Furthermore, the authors indicate that greater delays in adjusting the capital structure occur for countries with a worse macroeconomic environment (in the sense of GDP dynamics and higher interest rates, resulting in lower credit availability).

In conclusion, the above subchapter presents examples of empirical studies of the size of the cash holdings and the capital structure of companies, in which dynamic models estimated on panel data were used for modelling. The studies presented were selected in such a way as to indicate the significant diversity in the application of the aforementioned models, even in the context of the two corporate finance issues discussed. In addition, let us note that a summary of the parameter estimatesρ for the articles discussed is summarised in *Table 1* (it does not take into account the estimates obtained by the authors for the fixed effects estimator, random effects and pooled OLS estimator, as these methods are not appropriate for the problem under consideration).

Table 1. Range of magnitude of parameter estimates ρ in the empirical articles discussed.

Article	**Corporate finance issue**	**Range of parameter estimates obtained ρ**
A. Ozkan, N. Ozkan (2004)	Cash Holdings	0,395-0,526
P. Garcia-Teruel, P. Martinez-Solano (2008)	Cash Holdings	0,205-0,247
A. Shah (2011)	Cash Holdings	0,473-0,518
M. Bigelli, J. Sanchez-Vidal (2012)	Cash Holdings	-0,071-0,223[a]
A. Ozkan (2001)	Capital structure of	0,431
Z. Xu (2007)	Capital structure of	0,350-0,860
Ö. Öztekin, M. J. Flannery (2012)	Capital structure of the company	0,484-0,954[b]
W. Drobetz, D. C. Schilling, H. Schröder (2013)	Capital structure of the company	0,439-1,062

[a] parameter estimates ρ were found to be statistically indistinguishable from zero.

[b] the range of parameter estimates ρ is due to differences between the countries analysed by the authors and differences due to the use of different estimation methods.

Source: own study.

The very large variation in parameter estimates ρ within a single study is due in part to the adoption of a different range of explanatory variables or different assumptions about the adopted part of the database used for modelling, but the greatest variation $\hat{\rho}$ is due to the use of different estimation methods. This raises the question of which method is best (and under what conditions) to use and interpret for corporate finance studies.

1.2.2. Comparison of properties of estimators dedicated to dynamic panel models relation to corporate finance issues

The variation in parameter estimates ρ indicated above is another argument (after those presented in *Subchapter 1.1.2.*) for the validity of comparing the properties of estimators dedicated to dynamic panel models. In the literature, authors focus, as this paper does, on comparing the properties of estimators of the parameter with the dependent variable lagged by one period, as the most crucial parameter of the model (both in terms of economic interpretation and econometric validity). Research aimed at comparing the properties of the estimators of dynamic panel models in relation to issues in corporate finance topics will be discussed in the following subchapter.

The article by Flannery and Hankins[37] has, as it were, initiated a consideration of the comparative properties of estimators dedicated to dynamic panel models, in relation to the topic of corporate finance. The authors set out to conduct an in-depth analysis of estimators of dynamic panel models so that empirical researchers could select an appropriate method given the nature of the data at their disposal. Using data from the *Compustat* database, the authors analysed the optimal capital structure of non-financial companies that have had information in the aforementioned dataset for the last 30 years. In this way, a balanced panel was adopted for the study, while ignoring in the considerations the bias on the estimator of the parameter ρ, resulting from the data selection. The equation of the form(3) was taken as the main equation of the model,

[37] M. J. Flannery, K. W. Hankins, op. cit.

with the additional specification of the data generating process for the explanatory variables:

$$x_{kit} = \varphi_k x_{kit-1} + \alpha_1 y_{it-1} + \alpha_2 c_i + \omega_{it}, \tag{13}$$

where ω_{it} is derived from a normal distribution and $\varphi_k, \alpha_1, \alpha_2$ are the variable parameters in the simulations carried out in this paper. With this approach, it is possible to control the autocorrelation of the independent variables used in the subsequent simulations, but also to monitor their endogeneity and correlation with the individual effect. Furthermore, the researchers inflict ε_{it} as an *AR(2)* process, so that it is possible for them to control for the occurrence of first- and second-order autocorrelation in the purely random component. Two types of *Monte Carlo* simulations were run for the data-generating process thus defined: the independent case, where each x_{kit} is generated independently, and the case where x_{kit} is generated from a multivariate normal distribution based on a variance-covariance matrix derived from real data from the *Compustat* database. Then, within each type of simulation, *Monte Carlo* experiments were carried out considering the effect of the true size of the parameterρ, the length of the panel held and the endogeneity of the explanatory variables on the properties of the analysed estimators. The following estimation methods were adopted for consideration: pooled OLS estimator, fixed effects estimator (for comparison purposes only), Arellano-Bond estimator, Blundell-Bond estimator, *Long-difference Instrumental Variables Estimator* (for *k=4* and the highest possible within the meaning of the model discussed in *Subchapter 2.1.6.*) and *Least Square Dummy Variable Corrected Estimator*. All of these methods are discussed in detail in *Chapter 2*. As a result of their considerations, the authors indicate that the *Least Square Dummy Variable Corrected Estimator* is the best option when dealing with exogenous explanatory variables. Furthermore, when dealing with endogenous variables, the Blundell-Bond method has slightly better properties than the aforementioned estimator. In addition, the researchers point out that the *Long-difference Instrumental Variables Estimator* is very sensitive to the magnitude of the true parameter λ and can be characterised by significant biased. However, it is noteworthy that the authors only use the root mean square error to compare the properties of the estimators. Using this measure is not the best option for the type of variables analysed, due to the fact that they are limited to the interval $[0,1]$ (a more extensive discussion of prediction error measures is presented in *Appendix C*).

Liang and Kebin[38] considered the properties of estimators of dynamic models estimated on panel data, based on a different economic topic- transactional liquidity provision. The aim of this paper is to fill two gaps identified by the authors in the literature. First, the selection of explanatory variables to be used in the final model is to be done in an objective rather than arbitrary manner. To this end, two knowledge pooling methods were used: Bayesian knowledge pooling and *weighted-average least-square estimation*. Secondly, the authors want to find the optimal method for estimating the model by pointing out many methodological shortcomings in previous studies. The paper uses a balanced panel of 485 Chinese listed companies from 1999 to 2011. Companies engaged in financial activities, public utility companies, companies with negative value and bankrupt companies were excluded from the analysis. Information was again sourced from the *Compustat* database. The application of knowledge pooling methods made it possible to determine the seven most key factors relevant to the estimation of the final model. The authors then looked at comparing the properties of the Arellano-Bond estimators (both one-step and two-step), the Blundell-Bond estimators (both one-step and two-step), the *Long-difference Instrumental Variables Estimator* (for *k=4* in the sense of the model discussed in *Subchapter 2.1.6.*) and the suboptimal systemic generalised method of moments estimator (discussed in *Subchapter 2.1.4.*). To select the best model, the researchers conduct *Monte Carlo* simulations, generating the value of the explanatory variable based on the equation (eq. 3), using only real data (not assumed to be from the *AR(p)* process). The experiments were made dependent on the true size of the parameter ρ and the length of the panel held. The result of the work is that the estimator of the parameter ρ, with the best properties for all simulation scenarios, is the suboptimal system estimator of the generalised method

[38] C. Liang, D. Kebin, *The Dynamic Speed of Cash Holding Adjustment in Transition Economy: A New Approach and Evidences*, Guangdong University of Foreign Studies, 2014, On line, http://news.gdufs.edu.cn/Item/81450/paper2.pdf.

of moments. Furthermore, as the panel adopted for the analysis becomes longer, the properties (bias and efficiency) of all estimators considered improve. A significant strength of the paper is the comparison of multiple estimation methods; unfortunately, this is an unpublished paper in any regular journal. It was only presented at the Guangdong University of Foreign Studies conference (and published on the event website). As such, the study is not fully refined. At times, one can identify threads in it that need to be clarified, which in some aspects create an impression of arbitrariness (e.g. the way parameters are selected for estimation using the suboptimal systematic estimator of the generalised method of moments).

The optimal capital structure of a firm in the context of proposed estimator biased adjustments for dynamic panel models is considered by Zhou et al.[39]. The aim of the study is to propose biased adjustments for the most popular estimators of dynamic panel models. The authors decompose the biased of the considered estimators into three components: the biased due to the use of a given estimation method, the biased due to incorrect model specification and the biased due to the interaction of the two previously mentioned causes. The researchers point out that, so far, the most popular estimator based on biased correction (the *Least Square Dummy Variable Corrected Estimator* discussed in *Subchapter 2.1.6.*) takes into account and corrects only the part of it resulting from the estimation method used (for the method mentioned above, it is a fixed effects estimator). The contribution of the study is to extend this approach to correct the biased resulting from all three sources and, moreover, to make it applicable to all common estimation methods for dynamic panel models. The idea proposed by the researchers is based first on minimising the variance of a given estimator (this can result in an increase in biased) and then applying a linear biased correction, the form of which is indicated by the authors. The paper conducts extensive tests comparing two groups of estimation methods: the standard ones (pooled OLS estimator, fixed effects estimator, Arellano-Bond estimator, Blundell-Bond estimator, *Least Square Dummy Variable Corrected Estimator* and *Long-difference Instrumental Variables Estimator*) and identical methods to

[39] Q. Zhou, R. Faff, K. Alpert, op. cit.

the first group, except that after applying the biased correction proposed in the paper. The researchers conduct tests using both simulated data and information from 1965 to 2006 on US non-financial firms with assets greater than $10 million from the *Compustat* database. Furthermore, for the simulated data, they distinguish between a scenario in which the model is well specified, all variables are exogenous, independent and show no autocorrelation, and a counterfactual scenario. Finally, as a rule of thumb, for all methods, the parameter estimators ρ using the biased correction proposed in the paper have much better properties (the only case in which these estimators have minimally worse properties than their standard counterparts is in a well-specified model for simulated data when the true value of ρ is close to unity). In addition, the most appropriate estimation method for the issue under consideration in both groups of models is the two-step systematic Blundell-Bond estimator (standard and bias-adjusted versions, respectively). Let us note that this study does not consider extensively the problem of endogeneity of the dependent variables, in relation to the proposed bias adjustment. A direction for the development of the study could be to enrich it by addressing this issue.

A slightly different approach to bias adjustment is presented by Dnag, Kim and Shin[40]. The authors' objective is to investigate which of the existing estimators for dynamic panel models are the most appropriate and robust in the context of corporate finance research. Initially, the authors divide the estimation methods into two separate groups. The first includes methods based on instrumental variables and the generalised method of moments (the Anderson-Hsiao estimator - discussed in *Subchapter 1.2.1.*, the Arellano-Bond estimator, the Blundell-Bond estimator and the *Long-difference Instrumental Variables Estimator*). The second group, on the other hand, comprises estimators based on bias correction. These include: *Least Square Dummy Variable Corrected Estimator*, a fixed-effects estimator with a biased correction based on the bootstrap idea (so that no additional assumptions about the distribution of the variables and the correctness of the instruments are required), and a method called *Indirect Inference*, a method of estimation based on indirect inference. It assumes that

[40] V. A. Dnag, M. Kim, Y. Shin, op. cit.

an inverse bias function is found using simulation methods and then used to correct it. Any of the above-mentioned estimators can serve as the basis for the bias correction thus determined. The researchers then test the properties of the aforementioned methods against simulated data, where the explanatory variables and the purely random effect are derived from the *AR(1)* process, while the number of observations *N=100*, the number of panel waves *T=10*, and furthermore each simulation was performed 1,000 times. The different simulation scenarios include variation in the strength of the individual effect on the explained variable, variation in the strength of the effect of the other regressors on the dependent variable, the abrogation of the assumption of no autocorrelation of the random component, and the introduction of endogenous variables into the modelling. Conclusions from the analysis indicate that with an increase in the importance of the individual effect, the biased and *RMSE* for the Blundell-Bond and Anderson-Hsiao estimators increase significantly (unfortunately, the authors did not provide a possible reason for this). Furthermore, estimators based on biased adjustment, unlike their basic versions, are resistant to changing the strength of the effect of the regressors on the explanatory variable. Additionally, the absence of endogenous variables in the model improves the quality of the estimators for all methods, but the advantage of the bias-adjusted estimators over their standard counterparts is still observable. Finally, the authors conclude that for dynamic panel models of corporate finance it is better to use estimators based on biased correction, as they do not require the fulfilment of instrument validity assumptions and have slightly better properties in the sense of estimation precision and biased. The final step of the study is to apply the estimators discussed above to the issue of the capital structure of non-financial US firms, for which data from 1967 to 2006 were taken from the *Compustat* database. This analysis is only illustrative and, from an econometric point of view, adds nothing further to the study under discussion.

In the work presented above, the issue of the limitedness of the dependent variable was treated in a marginal way. This topic is the focus of Elsas and Florysiak in their comparisons[41]. The authors propose a new estimator that addresses

[41] E. Elsas, D. Florysiak, op. cit

the problem of two-sided censoring of the dependent variable. This is particularly relevant in the study of the capital structure of the companies that the authors analyse, because in addition to restricting the distribution of the dependent variable to the interval $[0,1]$, it is also significantly saturated at zero. The newly proposed method is called *Dynamic Panel Fractional Estimator* and is briefly discussed in *Subchapter 2.1.6.*. Based on this estimation method, the Arellano-Bond estimator, the fixed effects estimator and the pooled OLS estimator, *Monte Carlo* simulations were performed for comparison purposes. They were performed using real data from 1965 to 2008, on non-financial US companies, the source of which is the *Compustat* database. First, their replication process was performed, selecting the parameters of the data-generating process in such a way that the distribution of the resulting dependent variable was similar to the distribution of the dependent variable resulting from the real data. *Monte Carlo* simulations were then carried out, varying the assumptions about the true value of the parameter ρ. The researchers conclude that the use of the newly proposed estimation method makes it possible to reduce the bias on the parameter estimator ρ, by taking into account the fact that it is bounded. Let us note, however, that the reduction in bias compared to the Arellano-Bond estimator is not spectacular, but a significant gain from the new method is observed for ρ close to unity. For a more complete comparative picture, the range of estimators considered should still be extended to include the Blundell-Bond estimator, as its better properties compared to the Arellano-Bond estimator can be expected.

In summary, the comparison of the properties of dynamic panel model estimators in relation to the topic of corporate finance is a new issue in the literature. Hence, to the authors' knowledge, the studies presented in the above subchapter essentially exhaust the range of articles in this area. Considerations that present a slightly modified estimation method from the mainstream and compare several other methods with a newly proposed estimator have not been taken into account. Only studies whose real purpose is to compare the properties of the most popular estimators of dynamic panel models are presented.

Given the above, it is reasonable that the range of studies discussed does not include Polish-language items. Moreover, let us note that when it comes to, for example, the issue of cash holdings, domestic authors do not devote much attention to it. The dominant studies, usually in book form, refer to the overall issue of corporate liquidity, without distinguishing its transactional reserve. Moreover, Polish empirical studies on the subject in question focus on the statistical analysis of the phenomenon under consideration, not usually extending it to econometric modelling (especially based on dynamic models estimated on panel data).

Finally, let us further note that the studies discussed above comparing the properties of the estimators dedicated to dynamic panel models, in general, indicate one, possibly two, estimation methods that are, in the light of the results obtained, the most adequate and the best, for the issue under consideration. The problem is that these tend to be different methods between the articles presented. The issues in corporate finance for which dynamic models estimated on panel data are used are so broad that no single best estimation method should be identified within this entire class of issues (and possible data with different characteristics). Rather, narrower areas should be highlighted where the superiority of a certain estimation method can indeed be emphasised over others, regardless of the corporate finance topic under consideration and the dataset adopted for analysis. This type of guidance is proposed within the framework of this monograph in *Chapter 3*, which is somewhat of a change in approach, relative to studies presented in the literature.

In summary, this chapter presents the basic economic theories motivating the introduction of dynamics into corporate finance analyses. The focus here is on substitution theory and the theory of the hierarchy of funding sources in relation to the topics of cash holdings and corporate capital structure. In addition, the econometric advantages of modelling with panel data are presented and the reasons for the inapplicability of standard estimators for panel data (fixed and random effects and *pooled OLS*) for dynamic models are indicated. The scope of interest of this monograph - the properties of estimators for a parameter with a lagged dependent variable - is also motivated. The second part of the chapter presents examples of empirical studies (both purely economic and comparing the properties of estimators in relation to the topic of corporate finance), the results of which are a contribution to the consideration of the properties of estimators of dynamic panel models. Such an analysis, for the issue of transactional liquidity provision, is presented in *Chapter 3*, but is preceded in *Chapter 2* by a detailed discussion of estimation methods for dynamic models on panel data, together with their historical evolutionary path.

CHAPTER 2

METHODOLOGY FOR THE ESTIMATION OF DYNAMIC MODELS ON PANEL DATA

This subchapter discusses the estimation methodology for dynamic panel models, which provides both an overview of econometric methods and a description of the modelling techniques used for the subsequent empirical study in *Chapter 3* [42]. The order of the issues presented corresponds to the historical development of estimators for the group of models under consideration and, in this context, the following chapter has additional added value. In addition, the advantages and disadvantages of the various methods are indicated and basic tests from their diagnostic process are considered.

2.1. Estimation methods for dynamic panel models

This subchapter is devoted to presenting the development path of the estimation methodology for dynamic panel models, indicating the motivation for its evolution and presenting the advantages and disadvantages of the different estimation methods. The basis for the contents of the following subchapters are mainly book publications by Hsiao[43], Arellano[44], Baltagi[45], Mátyás and Sevestre[46]

[42] Not all of the methods presented in this subchapter are used in the later example empirical study, but for the sake of the logic of the argument they are discussed. For estimators not used in the modelling, the reason for this is explicitly noted in *the Chapter 3.*

[43] C. Hsiao, op. cit.

[44] M. Arellano, *Panel Data Econometrics*, Oxford: Oxford University Press, 2004, ISBN 0-19-924528-2.

[45] B. H. Baltagi, *Econometric Analysis of Panel Data*, The Atrium, Southern Gate, Chichester: John Wiley & Sons Ltd, 2005, ISBN 978-0-470-01456-1.

[46] L. Mátyás, P. Sevestre, *The Econometrics of Panel Data*, Berlin, Heidelberg: Springer-Verlag, 2008, ISBN: 978-3-540-75889-1.

and numerous articles by the developers of each estimator, which will be cited when discussing specific methods.

2.1.1. Origins of the development of dedicated estimation methods for dynamic panel models

As already mentioned in *Subchapter 1.1.2*, mainly due to the loss of consistency of classical estimators for static panel models, new estimation methods had to be developed to estimate dynamic models. Anderson and Hsiao are considered pioneers in considering this area. These researchers initially dealt with parameter estimation of dynamic panel models using the Maximum Likelihood Method[47] (hereafter also *ML*). In their first paper, they considered a basic autoregressive model of the form:

$$y_{it} = \rho y_{it-1} + c_i + \varepsilon_{it}, \text{dla } |\rho| < 1. \tag{14}$$

Paying particular attention to the properties of the fixed effects estimator and the *ML* estimator for this model, the authors conclude that a very important issue in the estimation process is the assumptions made about the initial conditions. The researchers identified various cases for the values of y_{i0}, considering for them the degree of complication of determining the maximum likelihood function. It turns out that when the values are non-random and independent of both individual effect and purely random error and have identical variance and mean, the reliability function takes a relatively simple form. It depends on the sample density and the initial values according to the equation be low:

$$f_i(y_{iT}, \dots, y_{i1}, y_{i0}) = f_i(y_{iT}, \dots, y_{i1} | y_{i0}) f_i(y_{i0}). \tag{15}$$

[47] T. W. Anderson, C. Hsiao, *Estimation of Dynamic Models with Error Components*, Journal of the American Statistical Association, 1981, vol. 76, pp. 598-606.

When the assumption of sameness of the mean or independence of the initial value is abrogated, the analytical form of the reliability function becomes very complicated and its determination is not a trivial task.[48]

Anderson and Hsiao[49] additionally compare the properties of the *Maximum Likelihood* estimator with the fixed effects estimator (for which the initial values are treated as fixed, but unknown). As discussed in *Subchapter 1.1.2.*, when the number of *T* panel waves is limited, the fixed effects estimator will never agree. Even at the current step of development of data collection techniques, in practice we are unlikely to have such long panels that the apparent contradiction of the $T \to +\infty$ condition can be questioned. Therefore, as the authors mention, the fixed effects estimator should not be used to estimate dynamic models on panel data. The maximum likelihood method is more appropriate in this respect. Namely, when the assumption of independence of non-random initial values from c_i and ε_{it} is met, the *Maximum Likelihood* estimator will be consistent for finite *T* and $N \to +\infty$[50].

These conclusions argue strongly in favour of using the *Maximum Likelihood* estimator rather than the fixed effects estimator to estimate dynamic models on panel data. In practice, however, the determination of the plausibility function for the problem under consideration is complicated and poses many difficulties, both in terms of computational tediousness and the problems of verifying assumptions about initial values. This paper therefore proposes the Instrumental Variables Method (hereafter also *IV*) as an alternative to *Maximum Likelihood* and the fixed effects estimator that is free of these inconveniences. However, it was necessary to make assumptions about the non-randomness of the individual effect, the absence of autocorrelation and the zero expectation value of the purely random error. The researchers then perform a two-sided differentiation of the *Equation 14,* which results

[48] C. Hsiao, op. cit., pp. 78-79. Let us additionally note that the analytical form of the credence function is not presented explicitly in this subchapter, as this issue deviates in some way from the main focus of the paper.

[49] T. W. Anderson, C. Hsiao, op. cit.

[50] Anderson and Hsiao point to another distinct case where such compliance will occur. Namely, when y_{i0} are observable, non-random and satisfy the condition $\lim_{n\to\infty} \frac{1}{\mathrm{N}} \sum_{i=1}^{N} y_{i0}^2 < +\infty$.

in the elimination of the individual effect. The resulting equation takes the following form:

$$(y_{it} - y_{it-1}) = \rho(y_{it-1} - y_{it-2}) + (\varepsilon_{it} - \varepsilon_{it-1}) \tag{16}$$

Note that Δy_{it-1} is correlated with the random component $\Delta\varepsilon_{it}$, so we are dealing with an endogeneity problem. Therefore, Anderson and Hsiao used IV to estimate the parameters of *Equation 16*. Δy_{it-2} and y_{it-2} have been identified as potential instruments. These are obviously correlated with Δy_{it-1}, while there is no correlation with $\Delta\varepsilon_{it}$, since it was assumed that for purely random error there is no autocorrelation. Note, however, that the decision to use Δy_{it-2} as an instrument shortens the panel by 3 waves, whereas using y_{it-2} shortens it by only 2 waves. Furthermore, as Arellano points out[51], the use of Δy_{it-2} as an instrument can result in a significant increase in the variance of model parameter estimates. Therefore recommended to use y_{it-2} as an instrumental variable for Δy_{it-1}. The main advantage of using IV for the problem under consideration is the independence of this estimation method from the initial values. Furthermore, the estimates obtained in this way are consistent when $T \to +\infty$ or $N \to +\infty$.

In the following paper[52], Anderson and Hsiao continue their discussion of the previously mentioned estimation methods, extending them to the case of a model with additional explanatory variables (in *k* counts):

$$y_{it} = \rho y_{it-1} + \sum_{\mathrm{k}} \beta_k x_{kit} + c_i + \varepsilon_{it}, \text{for } |\rho| < 1, \tag{17}$$

with the additional assumptions made being the absence of autocorrelation of purely random error $\mathbb{E}(c_i) = 0, \mathbb{E}(\varepsilon_{it}) = 0,$ and $\mathbb{E}(c_i\varepsilon_{it}) = 0.$ The authors

[51] M. Arellano, *A Note on the Anderson-Hsiao Estimator for Panel Data*, Economics Letters, 1989, Vol. 31, pp. 337 - 341.

[52] T. W. Anderson, C. Hsiao, *Formulation and Estimation of Dynamic Models Using Panel Data*, Journal of Econometrics, 1982, Vol. 18, pp. 47-87.

divide the paper into two main parts. Within each, as in the previous study, multiple cases are considered depending on the assumptions about the initial conditions. Due to the vastness of the content, the general idea of the method and the final conclusions will be discussed without breaking down all the cases proposed by Anderson and Hsiao[53]. The first part of the article presents the situation where the additionally introduced explanatory variables are constant over time. Estimation by the maximum likelihood method is then very complicated. Therefore, the possibility of using the results of the authors' previous work (for the basic autoregressive model) was noted. Well, the estimation of the parameter ρ can be obtained using the instrumental variables estimator for the model (14). Then inserting the fitted value $\hat{\rho}$ into the following equation:

$$\bar{y}_i - \rho\bar{y}_{i-1} = \sum_{\mathrm{k}} \beta_k x_{ki} + c_i + \bar{\varepsilon}_i, \tag{18}$$

where $\bar{y}_i, \bar{y}_{i-1}, \bar{\varepsilon}_i$ are the means (after time) of the variables y_{it}, y_{it-1} and ε_{it} respectively, parameter estimates β_k can be obtained using the least squares method.

In the second part of the paper, the authors additionally assume the introduced regressors to be time-varying. For the purpose of consideration under this assumption, the researchers define two models -state *dependence model* of the form:

$$\begin{cases} y_{it} = s_{it} + c_i \\ s_{it} = \rho s_{it-1} + \sum_{\mathrm{k}} \beta_k x_{kit} + \varepsilon_{it} \end{cases} \tag{19}$$

[53] ibid.

and *series correlation* character *model*:

$$\begin{cases} y_{it} = s_{it} + \sum_{\mathrm{k}} \beta_k x_{kit} + c_i \\ s_{it} = \rho s_{it-1} + \varepsilon_{it}, \end{cases} \tag{20}$$

where s_{it} denotes unobservable variables. Let us note that the difference between the two models considered is due to the different response of the explanatory variable to a shock in the value of the explanatory variables. Indeed, let us consider a one-period shock in the value of x_{ki} at time *t*. In the case of the state dependence model, this change will be visible in the distribution y_{it+1}, while the autocorrelation model is immune in this sense to the shock under consideration - it will not be visible in y_{it+1}. The dependencies in question are most easily observed by transforming the models (19) and (20) (by substituting the determined from the first equation of the system s_{it}, into the second equation of the model) to the following forms:

$$y_{it} = \rho y_{it-1} + \sum_{\mathrm{k}} \beta_k x_{kit} + F_i + \varepsilon_{it}, F_i = \rho c_i \tag{21}$$

for the state dependency model and:

$$y_{it} = \rho y_{it-1} + \sum_{\mathrm{k}} \beta_k x_{kit} - \rho \sum_{\mathrm{k}} \beta_k x_{kit-1} + F_i + \varepsilon_{it}, F_i = \rho c_i \tag{22}$$

for the autocorrelation model.

Anderson and Hsiao[54] initially propose the Maximum Likelihood method for the estimation of both models, considering its applicability depending on the variation of assumptions about initial conditions. Ultimately, however, the authors themselves acknowledge that the applicability of *Maximum Likelihood*

[54] ibid.

for the models in question is limited due to the difficulty of determining the credibility function. Therefore, they propose alternative methods - *IV* for the state dependence model and FGLS for the autocorrelation model. From the point of view of the present work, the most interesting model is (21), whose form coincides with the basic model considered in the paper. The application of instrumental variables to it, is preceded by the procedure of its two-sided differentiation. Finally, the first-differences model takes the form:

$$\begin{aligned}(\mathrm{y_{it}} - \mathrm{y_{it-1}}) = \rho(y_{it-1} - y_{it-2}) + \sum_{\mathrm{k}} \beta_k (x_{kit} - x_{kit-1}) \\ + (\varepsilon_{it} - \varepsilon_{it-1}),\end{aligned} \tag{23}$$

and the proposed instrumental variables for Δy_{it-1} are again y_{it-2} or Δy_{it-2}. The resulting parameter estimates of ρ and β_k are again consistent for $N \to +\infty$ or $T \to +\infty$.

The centrality of Anderson and Hsiao's contribution to the development of the methodology for estimating dynamic models on panel data is mainly manifested in two aspects. The first is to draw attention to the role of initial conditions in the estimation process, while the second (and much more important for the further development of the methodology) is to show how the individual effect can be eliminated by a two-sided differencing procedure of the initial equation. The *IV-based* method presented above produces consistent parameter estimates (when *N* is sufficiently large), but there are significant problems with the efficiency of the estimators obtained in this way. This situation has to do with not taking into account the varying structure of $\Delta\varepsilon_{it}$ and not using all available moment conditions in the estimation process. For this reason, this approach will not be used in the empirical study in *Chapter 3*, but it could not be omitted from the presentation of methodological material on the estimation of dynamic models on panel data, due to its indisputable contribution to this field. The proposal of more efficient estimators is the next step in the development of methodology in this area. These estimators are mostly based on the generalised method of moments (hereafter also *GMM*). Before presenting them, the main idea and assumptions of the *GMM* will be discussed

For the sake of the logic of the argument, let us mention that the classical method of moments is based on estimating theoretical moments using empirical moments from a sample. The application of this method is only possible when the number of unknown parameters is equal to the number of conditions imposed on the moments. For the *GMM*, the situation where such an equality does not occur is allowed, with the case where the number of conditions imposed on the moments is greater than the number of unknown parameters being particularly important.

The basic idea of the generalised method of moments is to select the model parameters in such a way that the empirical moments (from the sample) are matched to their theoretical values in the best possible way. In practice, this means that the *GMM* estimator of the parameter vector $\boldsymbol{\theta}$ should minimise the following objective (loss) function:

$$J(\boldsymbol{\theta}) = m(\boldsymbol{\theta})'\boldsymbol{W}m(\boldsymbol{\theta}), \tag{24}$$

where $\boldsymbol{\theta}$ denotes a vector of unknown parameters, $m(\boldsymbol{\theta})$ is a vector of conditions imposed on moments, *i.e.* a vector of sample-dependent functions (or instruments) and theoretical values of unknown model parameters in such a way that the condition $\mathbb{E}\big(m(\boldsymbol{y}, \boldsymbol{X}, \boldsymbol{\theta})\big) = \boldsymbol{0}$ is met. $\boldsymbol{W}$, on the other hand, represents a matrix of weights, which is discussed in more detail later in this subchapter.

In a situation analogous to the classical method of moments, when the number of estimated parameters is equal to the number of conditions imposed on the moments, the estimates should minimise the objective function (24). However, the question remains what if the number of conditions imposed on the moments is greater than the number of estimated parameters. Well, it may not be possible to satisfy them all at once. A way to solve this inconvenience is to indicate which of the conditions of moments are less and which are more important to us. For this purpose, the weight matrix $\boldsymbol{W}$ is used in *GMM.* By assumption, it is a matrix with a row equal to the number of moment conditions, symmetric and positively defined, hence its simplest and most intuitive choice could be the unitary matrix. However, such an approach may induce inefficiencies in the obtained estimators, hence in practice it is usual to use an iterative method to determine the $\boldsymbol{W}$ matrix. It involves taking the unitary matrix as the initial weight matrix $\boldsymbol{W}_1$ (it could also be any other positively symmetric matrix), and then minimising the objective function to determine the initial estimator $\widehat{\boldsymbol{\theta}}_1$. In the next step, the asymptotic autocovariance matrix of the vector parameters $\widehat{\boldsymbol{\theta}}_1$ is taken as the new weights matrix $\boldsymbol{W}_2$. With its help, the estimator $\widehat{\boldsymbol{\theta}}_2$ minimising the newly created objective function is determined again. Analogous iterations can be repeated further until a satisfactorily low value of the objective function is obtained. In practice, however, only 2 or 3 iterations are usually carried out. A quantified notation of the procedure described above is as follows:

$$\text{step 1}\begin{cases} \boldsymbol{W}_1 = \boldsymbol{I} \\ \widehat{\boldsymbol{\theta}}_1 = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, m(\boldsymbol{\theta})'\boldsymbol{W}_1 m(\boldsymbol{\theta}), \end{cases} \tag{25}$$

$$\text{step 2}\begin{cases} \boldsymbol{W}_2 = f(\widehat{\boldsymbol{\theta}}_1) \\ \widehat{\boldsymbol{\theta}}_2 = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, m(\boldsymbol{\theta})'\boldsymbol{W}_2 m(\boldsymbol{\theta}). \end{cases} \tag{26}$$

The second extreme case in *GMM* estimation is that the number of estimated unknown parameters is larger than the number of conditions imposed on the moments. It is then possible to construct additional conditions by means of instrumental variables (*a.k.a.* instruments), *i.e.* variables z_t, which are correlated with the explanatory variable for which they are an instrument and uncorrelated with purely random error. Then, with their help, it is possible to create additional conditions of moments of the following form:

$$m_i(\boldsymbol{y}, \boldsymbol{X}, \boldsymbol{z}, \boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T}(y_t - \theta_t x_t) z_t. \tag{27}$$

This approach is of particular relevance when estimating dynamic models on panel data, as the design of the individual estimators for this group of models relies on an appropriate choice of instruments. This is also one of the main aspects that differentiate the different estimation methods discussed in the following subchapters.

To summarise the discussion of the *GMM*, which forms the basis of most estimators for dynamic panel models, it should be pointed out that it has many advantages. The most important of these are that there are no restrictive assumptions about the distribution of the random component, that heteroskedasticity is allowed, and that when considering economic problems, the conditions of moments are often derived from the theory or the form of the model itself. However, it should be pointed out,, that the problem of weak instruments, *i.e.* a situation in which the instrumental variable used is very weakly correlated with the original explanatory variable, is possible with *GMM*. This may be the reason for the loss of consistency of the *GMM* estimator. In addition, the asymptotically normal distribution that the *GMM* estimator has may be inadequate when used for small samples. Nevertheless, due to its advantages, this method is the foundation of dedicated methods for estimating parameters of dynamic models on panel data, which will be discussed in the following subchapters.

2.1.2. Arellano-Bond first difference estimator

Historically, the earliest significant method for estimating dynamic models on panel data that uses *GMM* is the first-difference estimator proposed by Arellano and Bond[55]. Initially, analogous to Anderson and Hsiao[56], the authors consider an autoregressive model, as defined in (14), but additionally make the assumptions that there is no autocorrelation of purely random error and that $\mathbb{E}(c_i) = 0, \mathbb{E}(\varepsilon_{it}) = 0, \mathbb{E}(c_i\varepsilon_{it}) = 0$ and $\mathbb{E}(y_{i1}\varepsilon_{it}) = 0$ (for $i = 1 \dots N$; $t = 2 \dots T$) are satisfied. The researchers also perform a two-sided differential treatment of the initial equation to eliminate the individual effect and in this sense

[55] M. Arellano, S. Bond, *Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations*, The Review of Economic Studies, 1991, Vol. 58, pp. 277-297.

[56] T. W. Anderson, C. Hsiao, *Estimation of Dynamic Models...*, op. cit.

benefit from the contributions of the work discussed in the previous chapter. However, they do not use IV to derive parameter estimates, but use GMM.

Arellano and Bond base their proceeding on the observation that the use of the generalised method of moments yields consistent estimators (additionally characterised by a relatively small variance) for $N \to +\infty$ and T finite, with the conditions imposed on the moments taking the form of:

$$\mathbb{E}(\Delta\varepsilon_{it} y_{it-s}) = 0, t = 3 \dots T, t > s \geq 2. \tag{28}$$

Let us consider the correctness and validity of this definition of the conditions of moments. Well, for $t = 3$ the condition (28) takes the form:

$$\mathbb{E}\big((\varepsilon_{i3} - \varepsilon_{i2}) y_{i1}\big) = 0. \tag{1}$$

Adding both sides to the equation (1) $(\varepsilon_{i3} - \varepsilon_{i2})$ we get:

$$\mathbb{E}\big((\varepsilon_{i3} - \varepsilon_{i2}) y_{i1} + (\varepsilon_{i3} - \varepsilon_{i2})\big) = (\varepsilon_{i3} - \varepsilon_{i2}). \tag{30}$$

From the assumptions of the model we have that $\mathbb{E}(y_{i1}\varepsilon_{it}) = 0$ for any i and $t > 2$, and additionally based on (14) $\varepsilon_{it} = y_{it} - \rho y_{it-1} - c_i$, therefore substituting the above into the equation (30) we finally get:

$$(y_{i3} - y_{i2}) = \rho(y_{i2} - y_{i1}) + (\varepsilon_{i3} - \varepsilon_{i2}). \tag{31}$$

The instrument for $(y_{i2} - y_{i1})$ in the above equation can obviously be y_{i1} due to its strong correlation with Δy_{i2} and lack of correlation with $\Delta\varepsilon_{i3}$.

Proceeding by analogy, for $t = 4$ we obtain the equation:

$$(y_{i4} - y_{i3}) = \rho(y_{i3} - y_{i2}) + (\varepsilon_{i4} - \varepsilon_{i3}). \tag{32}$$

However, in this case the intuitive instrument $(y_{i3} - y_{i2})$ is both y_{i2} and y_{i1}, due to their apparent lack of correlation with $\Delta\varepsilon_{i4}$ and the presence of such a correlation with Δy_{i3}.

Continuing the discussion in the same way, it can be concluded that, for a fixed *t*, the appropriate instruments of the resulting equation: $(y_{it} - y_{it-1}) = \rho(y_{it-1} - y_{it-2}) + (\varepsilon_{\mathrm{it}} - \varepsilon_{it-1})$, will be the variables $y_{i1}, \dots, y_{it-2}$.

Let us now turn to an alternative way of writing down the most important of the previously considered equations, which is intended to facilitate the following considerations. Let us therefore define:

$$\boldsymbol{Z}_i = \begin{bmatrix} [y_{i1}] & \boldsymbol{0} & \dots & \boldsymbol{0} \\ 0 & [y_{i1}, y_{i2}] & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ 0 & \dots & \boldsymbol{0} & [y_{i1}, \dots, y_{iT-2}] \end{bmatrix}, \tag{33}$$

that is, the successive rows of the matrix $\boldsymbol{Z}_i$ on its main diagonal (in the sense of a block-diagonal matrix) contain instruments that correspond to a period *t* equal to the number of the row in which they are located (the designation $[y_{i1}, \dots, y_{it}]$ corresponds to defining the *t* columns of the matrix $\boldsymbol{Z}_i$ having on the *t^th^* line the corresponding instrument, and in the remaining rows zeros). Therefore, the dimension of the matrix $\boldsymbol{Z}_i$ is $m \times n$, where $m = T - 2$, and $n = \frac{1}{2}(T-1)(T-2)$. In addition, let us define:

$$\Delta\boldsymbol{\varepsilon}_i = \begin{bmatrix} \varepsilon_{i2} - \varepsilon_{i1} \\ \dots \\ \varepsilon_{iT} - \varepsilon_{iT-1} \end{bmatrix}, \tag{34}$$

which is a matrix of dimension $m \times 1$.

Then, using (33) and (34) the notation of the conditions imposed on the moments defined in (28), takes the matrix form $\mathbb{E}(\boldsymbol{Z}_i^T \Delta\boldsymbol{\varepsilon}_{it}) = \boldsymbol{0}$. Furthermore, the basic model (16) can be rewritten as:

$$\Delta \boldsymbol{y} = \Delta \boldsymbol{y}_{-1}\rho + \Delta \boldsymbol{\varepsilon}, \tag{35}$$

where $\Delta \boldsymbol{y} = [\Delta \boldsymbol{y}_1^T, \dots, \Delta \boldsymbol{y}_N^T]^T$ for $\Delta \boldsymbol{y}_i$ defined analogously to $\Delta \boldsymbol{\varepsilon}_i$ for $i > 0$, and $i = -1$ defined as $\Delta \boldsymbol{y}_{-1} = \left[\Delta \boldsymbol{y}_{1,-1}^T, \dots, \Delta \boldsymbol{y}_{N,-1}^T\right]^T$ (where $\Delta \boldsymbol{y}_{i,-1} = [y_{i3} - y_{i2}, \dots, y_{iT} - y_{iT-2}]^T$). In addition, $\Delta \boldsymbol{\varepsilon} = [\Delta \boldsymbol{\varepsilon}_1, \dots, \Delta \boldsymbol{\varepsilon}_{\mathrm{N}}]^T$.

A further part of the estimation procedure proposed by Arellano and Bond[57] is to use the generalised method of moments to find the final parameter estimate ρ. Multiplying both sides of the equation (35) by the instrument matrix $\boldsymbol{Z} = [\boldsymbol{Z}_1^T, \dots, \boldsymbol{Z}_N^T]^T$ yields the final equation:

$$\boldsymbol{Z}^T \Delta \boldsymbol{y} = \boldsymbol{Z}^T \Delta \boldsymbol{y}_{-1}\rho + \boldsymbol{Z}^T \Delta \boldsymbol{\varepsilon}, \tag{36}$$

with which the form of the objective function $J(\rho)$ for minimisation is determined:

$$J(\rho) = \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{Z}_i^T\left(\Delta \boldsymbol{y}_i - \rho \Delta \boldsymbol{y}_{i,-1}\right)\right]^T \boldsymbol{W} \left[\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{Z}_i^T\left(\Delta \boldsymbol{y}_i - \rho \Delta \boldsymbol{y}_{i,-1}\right)\right], \tag{37}$$

where $\boldsymbol{W}$ is a positively defined and symmetric weight matrix.

Let us note that the definition of the above objective function takes place because usually the number of conditions imposed on the moments is greater than the number of unknown parameters. Moreover, the matrix of weights $\boldsymbol{W}$ must be defined *ex-ante*. Arellano and Bond[58] propose its form as follows (instead of adopting the identity matrix):

$$\boldsymbol{W} = \left(\frac{1}{N}\sum_{i=1}^{N} \boldsymbol{Z}_i^T \boldsymbol{H} \boldsymbol{Z}_{\mathrm{i}}\right)^{-1}, \tag{38}$$

[57] M. Arellano, S. Bond, op. cit.

[58] ibidem, pp. 279.

where:

$$\boldsymbol{H} = \begin{bmatrix} 2 & -1 & 0 & \dots \\ -1 & 2 & \ddots & 0 \\ 0 & \ddots & \ddots & -1 \\ \vdots & 0 & -1 & 2 \end{bmatrix}. \tag{39}$$

Finally, the estimator of the parameter ρ, determined by the above method, called the one-step Arellano-Bond first difference estimator[59], takes the form:

$$\hat{\rho}_{AB} = \left[\left(\sum_{i=1}^{N} \Delta\boldsymbol{y}_{i,-1}^{T}\boldsymbol{Z}_i\right)\boldsymbol{W}\left(\sum_{i=1}^{N}\boldsymbol{Z}_i^{T}\Delta\boldsymbol{y}_{i,-1}\right)\right]^{-1}\left[\left(\sum_{i=1}^{N} \Delta\boldsymbol{y}_{i,-1}^{T}\boldsymbol{Z}_i\right)\boldsymbol{W}\left(\sum_{i=1}^{N}\boldsymbol{Z}_i^{T}\Delta\boldsymbol{y}_{i,-1}\right)\right]. \tag{40}$$

According to the idea of the generalised method of moments, the estimator $\hat{\rho}_{AB1}$ may not be efficient, due to the assumed weight matrix $\boldsymbol{W}$. For this reason, Arellano and Bond[60] also propose a two-step estimator, which is obtained from the one-step estimator after replacing the matrix $\boldsymbol{W}$ by its asymptotically efficient and consistent form estimator (iterative procedure for the generalised method of moments, described in the previous subchapter):

$$\boldsymbol{W}_2 = \left(\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{Z}_i^{T}\Delta\hat{\boldsymbol{\varepsilon}}_i\Delta\hat{\boldsymbol{\varepsilon}}_i^{\mathrm{T}}\mathbf{Z}_{\mathrm{i}}\right)^{-1}, \tag{41}$$

where $\Delta\hat{\boldsymbol{\varepsilon}}_i$ is a matrix defined analogously to that in the formula (34), but based on the residuals from the one-step Arellano-Bond estimation procedure. Let us note that the use of the two-step method (for both the Arellano-Bond estimator and the systematic estimator of the generalised Blundell-Bond method

[59] The full name of the estimator is the Arellano-Bond first-difference estimator, but if this does not lead to confusion, it will sometimes be referred to as simply the Arellano-Bond estimator in the remainder of this paper.

[60] M. Arellano, S. Bond, op. cit.

of moments, which is discussed in *Subchapter 2.1.3.*) is intended to improve the efficiency of the estimator. However, this is not synonymous with a concomitant reduction in bias. In particular, it is possible that a one-step estimator will have a lower bias than a two-step estimator.

For the method in question, no restrictive assumptions are generally made, which is a definite advantage. The only condition that must be met for the consistency of the Arellano-Bond first difference estimator is the absence of second-order autocorrelation $\Delta\varepsilon_{it}$. The authors propose a special statistical test to verify this assumption. It is presented in *Subchapter 2.3.* Furthermore, if ε_{it} has a constant variance over time (it is homoskedastic), then the optimal weight matrix is given by the formula (38). It is this relationship that dictated the choice of the $\boldsymbol{H}$ matrix, such as in the formula (39). At the same time, in such a situation, the one-step Arellano-Bond first-difference estimator is equivalent to the two-step estimator, but when ε_{it} is heteroskedastic, the estimator of the two-step method will be more efficient.

The methodology considered by Arellano and Bond[61] is quite simply generalised to the case of the model (eq. 3) - with additional explanatory variables $\boldsymbol{x}_{it}$. Clearly the key issue in this context is the nature of the individual variables in the vector $\boldsymbol{x}_{it}$. They are divided into strictly exogenous, predetermined variables and endogenous variables. The first group are variables that are uncorrelated with future, current and past values of the purely random component. The second is a group of variables uncorrelated with the current values of the purely random component, but correlated with its past values. The last group, on the other hand, is represented by variables correlated with the current and past values of the purely random error, however, there is no correlation with its future values.

The authors divide the considerations into two disjoint cases. For the first one, it is assumed that the additionally introduced explanatory variables are correlated with the individual effect c_i. Then Arellano and Bond[62] propose additional instruments for the equation under consideration on the differences in successive periods t. For strictly exogenous variables these will be their first differences, while for predetermined and endogenous variables these will be the

[61] ibid.
[62] ibid.

lagged levels of these variables. Indeed, let us assume that $\boldsymbol{x}_{it}$ are exogenous. Then, by definition, we have $\mathbb{E}\boldsymbol{x}_{it}\varepsilon_{is} = \mathbf{0}$, and consequently it is true that $\mathbb{E}\boldsymbol{x}_{it}\Delta\varepsilon_{is} = \mathbf{0}$, and in this way the additional conditions imposed on the moments could be determined, from which it follows that for the equation of first differences obtained from the model (eq. 3) the appropriate, additional instruments would be $\boldsymbol{x}_{i1}, \dots, \boldsymbol{x}_{it}$. It would then be necessary to extend the matrix (33) with additional T columns for each period. This would result in a very large increase in the order of the matrix $\boldsymbol{Z}_i$, which could give rise to computational performance problems. An alternative solution, however, which causes a reduction in efficiency (but very slight), is to use $\Delta\boldsymbol{x}_{it}$ as instruments for $\boldsymbol{x}_{it}$. Then the conditions of moments take the form:

$$\mathbb{E}\Delta\boldsymbol{x}_{it}\Delta\varepsilon_{is} = \mathbf{0}, \tag{42}$$

whereas the instrument matrix will only have an order higher by $T - 2$ and can be written as:

$$\boldsymbol{Z}_i = \begin{bmatrix} [y_{i1}, \Delta\boldsymbol{x}_{i2}^T] & \mathbf{0} & \cdots & \mathbf{0} \\ 0 & [y_{i1}, y_{i2}, \Delta\boldsymbol{x}_{i3}^T] & \mathbf{0} & \mathbf{0} \\ \vdots & \mathbf{0} & \ddots & \mathbf{0} \\ 0 & \cdots & \mathbf{0} & [y_{i1}, \dots, y_{iT-2}, \Delta\boldsymbol{x}_{iT}^T] \end{bmatrix}. \tag{43}$$

Now suppose that the variables $\boldsymbol{x}_{it}$ are predetermined or endogenous. Then, by definition, we have that $\mathbb{E}\boldsymbol{x}_{it}\varepsilon_{is} \neq \mathbf{0}$ for $t >= s$ and that $\mathbb{E}\boldsymbol{x}_{it}\varepsilon_{is} = \mathbf{0}$ for $t < s$, while additional conditions of moments can be defined as:

$$\mathbb{E}\boldsymbol{x}_{it-s}\Delta\varepsilon_{is} = \mathbf{0}, \text{dla } t > 3, s \geq 2. \tag{44}$$

Writing the equation on the differences for the model (eq. 3) and taking $t = 3$:

$$y_{i3} - y_{i2} = \rho(y_{i2} - y_{i1}) + (\boldsymbol{x}_{i3}^T - \boldsymbol{x}_{i2}^T)\boldsymbol{\beta} + (\varepsilon_{i3} - \varepsilon_{i2}), \tag{45}$$

we can see that the relevant instruments for $\Delta\boldsymbol{x}_{i3}^T$ are $\boldsymbol{x}_{i2}^T$ and $\boldsymbol{x}_{i1}^T$, as they are obviously correlated with $\Delta\boldsymbol{x}_{i3}^T$, while we do not identify their correlation with $\Delta\varepsilon_{i3}$. Continuing the analogous path of considerations for subsequent periods t, we obtain that the instruments of $\Delta\boldsymbol{x}_{it}$ for a given t are $\boldsymbol{x}_{i1}^T, \dots, \boldsymbol{x}_{iT-1}^T$, and consequently the instrument matrix $\boldsymbol{Z}_i$ takes the form:

$$\boldsymbol{Z}_i = \begin{bmatrix} [y_{i1}, \boldsymbol{x}_{i1}^T, \boldsymbol{x}_{i2}^T] & \boldsymbol{0} & \dots & \boldsymbol{0} \\ 0 & [y_{i1}, y_{i2}, \boldsymbol{x}_{i1}^T, \boldsymbol{x}_{i2}^T, \boldsymbol{x}_{i3}^T] & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \boldsymbol{0} & \ddots & \boldsymbol{0} \\ 0 & \dots & \boldsymbol{0} & [y_{i1}, \dots, y_{iT-2}, \boldsymbol{x}_{i1}^T, \dots, \boldsymbol{x}_{iT-1}^T] \end{bmatrix}. \quad (46)$$

In the general case where we are simultaneously dealing with strictly endogenous, predetermined and exogenous variables within a single model, the instrument matrix $\boldsymbol{Z}_i$ will be a suitable combination of the matrices (43) and (46).

The second main case considered by Arellano and Bond[63] is when the variables $\boldsymbol{x}_{it}$ (or, more precisely, some of them) are not correlated with the individual effect c_i. The authors then propose to decompose $\boldsymbol{x}_{it}$ into two subsets: variables that are correlated with c_i (let us denote them by $\boldsymbol{x}_{it,A}$) and variables that do not show such a correlation (let us denote them by $\boldsymbol{x}_{it,B}$). For the second group, the determination of the relevant instruments is based on the idea of the Hausman-Taylor estimator, however, this will not be discussed in detail in this subchapter, as in the case of corporate finance research the explanatory variables are correlated with the individual effect. Let us only note that, in the absence of such correlation, the instrument matrix will take the following modified form:

$$\boldsymbol{Z}_i^{H-T} = \begin{bmatrix} \boldsymbol{Z}_i & \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & [\boldsymbol{x}_{i1,B}^T, \boldsymbol{x}_{i2,B}^T] & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{x}_{i3,B}^T & \ddots & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{x}_{iT,B}^T \end{bmatrix}, \quad (47)$$

[63] ibid.

where $\boldsymbol{Z}_i$ is given by (43) (46) or a combination of both (depending on the nature of the explanatory variables).

The further part of the estimation procedure proposed by Arellano and Bond[64], as in the case of the autoregressive model, consists in using the generalised method of moments to find the final estimator, and its form is analogous to (40), taking into account the presence of additional variables $\boldsymbol{x}_{it}$ and a modified matrix $\boldsymbol{Z}_i$ for the case of these additional regressors. The final estimator takes the form:

$$\begin{bmatrix}\hat{\rho}\\ \hat{\boldsymbol{\beta}}\end{bmatrix}_{AB} = \\ = \left[\left(\sum_{i=1}^{N}\left(\Delta\boldsymbol{y}_{i,-1}\left[\Delta\boldsymbol{y}_{i,-1}\ \Delta\boldsymbol{x}_{it}\right]^{T}\right)^{\mathrm{T}}\boldsymbol{Z}_i\right)\boldsymbol{W}\left(\sum_{i=1}^{N}\boldsymbol{Z}_i^T\Delta\boldsymbol{y}_{i,-1}\left[\Delta\boldsymbol{y}_{i,-1}\ \Delta\boldsymbol{x}_{it}\right]^{T}\right)\right]^{-1}\cdot \\ \cdot\left[\left(\sum_{i=1}^{N}\left(\Delta\boldsymbol{y}_{i,-1}\left[\Delta\boldsymbol{y}_{i,-1}\ \Delta\boldsymbol{x}_{it}\right]^{T}\right)^{\mathrm{T}}\boldsymbol{Z}_i\right)\boldsymbol{W}\left(\sum_{i=1}^{N}\boldsymbol{Z}_i^T\Delta\boldsymbol{y}_{i,-1}\left[\Delta\boldsymbol{y}_{i,-1}\ \Delta\boldsymbol{x}_{it}\right]^{T}\right)\right] \tag{48}$$

In summary, Arellano and Bond[65] extended the approach of Anderson and Hsiao[66] so that the performance of the obtained estimators could be improved. The main improvement was the use of the generalised method of moments instead of the instrumental variables method. Let us note, however, that when dealing with the problem of weak instruments (low correlation between instruments and explanatory variables), the obtained estimators can lose efficiency and be biased. This problem determined the further development of the methodology, which is discussed in the next subchapter.

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[64] ibid.

[65] ibid.

[66] T. W. Anderson, C. Hsiao, *Estimation of Dynamic Models...*, op. cit.

2.1.3. Systematic estimator of the generalised Blundell-Bond method of moments

The Arellano-Bond first-difference estimator, although a certain refinement of the approach of Anderson and Hsiao[67], is not free of drawbacks. Well, as mentioned earlier, when dealing with weak instruments the Arellano-Bond estimator can be biased and inefficient. The problem of low correlation between explanatory variables and their instruments is generally encountered in two cases, when the value of the parameterρ is close to unity and when the ratio of the variance of the individual effect c_i to the variance of the purely random error ε_{it} is relatively high. These cases were identified and considered by Blundell and Bond.[68]

In the study, the authors focused on a standard autoregressive model of the form (14), adding a technical assumption on the initial condition y_{i1} of the following form:

$$\mathbb{E}y_{i1}\varepsilon_{it} = 0 \text{ dla } i = 1, \dots, N; t \geq 2. \tag{49}$$

In addition, Blundell and Bond's initial discussion[69] was limited to the case of$T = 3$, arguing, however, that all conclusions are also true for any T and model (eq. 3) - with additional explanatory variables, and the focus on the autoregressive model is due to a desire to keep the notation simple.

We will therefore consider the model (14) for $T = 3$. Then we have only one condition imposed on the moments, taking the form (1), so there is equality of the number of unknown parameters and conditions of moments, and consequently ρ is uniquely identified. The procedure for determining the one-step Arellano-Bond estimator reduces to determining the IV estimator from the following equation (formed by subtracting y_{i1} on both sides from (14) for the chosen T):

[67] ibid.

[68] R. Blundell, S. Bond, *Initial Conditions and Moment Restrictions in Dynamic Panel Data Models*, Journal of Econometrics 87, 1998, pp. 115-143.

[69] ibid.

$$y_{i2} - y_{i1} = (\rho - 1)y_{i1} + c_i + \varepsilon_{i2}. \tag{50}$$

Blundell and Bond showed that, in the case under consideration, the biased of the Arellano-Bond estimator of the parameter $\rho - 1$ is negative and increases significantly for $\rho \to 1$ and for a high quotient value of $\frac{\sigma_c^2}{\sigma_\varepsilon^2}$, where σ_c^2 denotes the variance of the individual effect, while σ_ε^2 denotes the variance of the purely random error. Note that the value of the $\hat{\rho} - 1$ bias is equal to the value of the $\hat{\rho}$ bias, and in this sense the above conclusions apply to the inference of the $\hat{\rho}$ bias. Indeed, $b(\hat{\rho} - 1) = \mathbb{E}(\hat{\rho} - 1) - (\rho - 1) = \mathbb{E}\hat{\rho} - \rho = b(\hat{\rho})$. Moreover, in the above situations, as the authors mention, an increase in bias is accompanied by a significant decrease in the efficiency of the Arellano-Bond estimator. The problem of weak instruments is responsible for this state of affairs.

This determined the need to improve the Arellano-Bond first-difference estimator so that the negative effects of weak correlation of instruments with explanatory variables are limited. A new estimation method was proposed by Blundell and Bond[70], and its idea is based on estimating a system of equations on increments (as in the Arellano-Bond estimator) and on levels (be for e variation – equation (14)). The very idea of using the equation on levels comes from Arellano and Bover[71], while Blundell and Bond combine it with the equation on increments, in this sense capturing all the added value of Arellano and Bover's considerations in their study, which explains the fact that this estimation method has not entered the canon of methods for estimating dynamic models on panel data. Consequently, the Arellano and Bover estimator is also not discussed in this paper.

Blundell and Bond[72] set out additional conditions on moments and instruments for the new equation. However, it is noted that for this to be possible an additional condition on the initial values[73] added to the assumptions of the initial autoregressive model considered by Arellano and Bond is needed

[70] ibid.

[71] M. Arellano, O. Bover, *Another look at the instrumental variable estimation of error-components models*, Journal of Econometrics, 1995, Vol. 68, pp. 29-51.

[72] R. Blundell, S. Bond, op. cit.

[73] Blundel and Bond refer to the quoted condition as *mild stationarity restriction*.

(these assumptions are defined in the introduction to *Subchapter 2.1.2.*). It takes the following form:

$$\mathbb{E}\left(y_{i1} - \frac{c_i}{1-\rho}\right)c_i = 0. \tag{51}$$

In practice, the above condition means that y_{i1} is stationary about the mean (i.e. we do not identify a deterministic trend in it) and in empirical studies it is treated rather as a fulfilled technical condition.

Remaining all along in the consideration for $T = 3$ let us show that the equation (51) is equivalent to the equation:

$$\mathbb{E}(c_i \Delta y_{i2}) = 0 \; for \; i = 1, \dots, N. \tag{52}$$

Well, let us assume without reducing generality that:

$$y_{i1} = \frac{c_i}{1-\rho} + \xi_{i1}, \tag{53}$$

where $\xi_{i1} = c_i + \varepsilon_{i1}$, and this equation is a transformation of the basic equation of the model under consideration, based on the assumption used in the literature $y_{i1} = y_{i0}$[74]. Then substituting (53) into (50) we have:

$$y_{i2} - y_{i1} = (\rho - 1)\left(\frac{c_i}{1-\rho} + \xi_{i1}\right) + c_i + \varepsilon_{i2} = (\rho - 1)\xi_{i1} + \varepsilon_{i2}. \tag{54}$$

Further inserting the obtained into (eq. (52) we get: $\mathbb{E}(c_i[(\rho - 1)\xi_{i1} + \varepsilon_{i2}]) = 0 \Rightarrow (\rho - 1) \cdot \mathbb{E}(\xi_{i1} c_i) + \mathbb{E}(c_i \varepsilon_{i2}) = 0 \Rightarrow (\rho - 1)\mathbb{E}(\xi_{i1} c_i) = 0 \Rightarrow \mathbb{E}(\xi_{i1} c_i) = 0$, where the penultimate result is a consequence of the model's assumption of no

[74] J. M. Wooldridge, *Econometric Analysis of Cross Section and Panel Data*, Cambridge, Massachusetts: The MIT Press, 2010, ISBN: 978-0-262-23258-6

correlation between the individual effect and the purely random component. Thus, we have shown that the condition (52) is equivalent to the condition (51), with the only requirement being that y_{i1} satisfies the assumption of weak stationarity: $\mathbb{E}(y_{i1}|c_i) = \frac{c_i}{1-\rho}$.

This reasoning can be generalised to the case of any $t \geq 3$ and then we obtain the following additional $T-2$ conditions imposed on moments:

$$\mathbb{E}\xi_{it}\Delta y_{it-1} = 0 \text{ dla } i = 1, \ldots, N; t \geq 3, \tag{55}$$

where $\xi_{it} = c_i + \varepsilon_{it}$. It follows that Δy_{it-1} (lagged first differences of the explained variable) can be used as instruments for the equation in levels. In addition, other possible instruments are Δy_{it-s}, where $s > 1$, but these instruments are the same as the instruments resulting from the equation (28). However, the observation that Δy_{it-s} can be used as an instrumental variable will be more relevant when extending the output model with additional explanatory variables $\boldsymbol{x}_{it}$. The systematic estimator of the generalised Blundell-Bond method of moments[75] uses both the conditions imposed on the moments for the equation on differences - (28), and the equation on levels - (eq. (55), whereby for the equation on levels the instruments are lagged first differences of the explanatory variable. They can be used in this capacity because despite the correlation of c_i with y_{it}, according to (52) there is no correlation between c_i and Δy_{it}.

For the purposes of the analogous notation of the conditions of moments in matrix form as in *Subchapter 2.1.2.*, let us define:

$$\boldsymbol{Z}_i^{BB} = \begin{bmatrix} \boldsymbol{Z}_i & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Delta y_{i2} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \Delta y_{i3} & \ddots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \Delta y_{iT-1} \end{bmatrix}, \tag{56}$$

where $\boldsymbol{Z}_i$ is given by (eq. (33) and:

[75] This estimator is also referred to in this paper as the Blundell-Bond estimator.

$$\boldsymbol{\xi}_i^{BB} = [\Delta\varepsilon_{i3}, \dots, \Delta\varepsilon_{iT}, \xi_{i3}, \dots, \xi_{iT}]^T, \tag{57}$$

where $\xi_{it} = c_i + \varepsilon_{it}$. Then the additionally obtained conditions of moments (55) will be in matrix form:

$$\mathbb{E}\left(\boldsymbol{Z}_i^{BB^T}\boldsymbol{\xi}_i^{BB}\right) = \mathbf{0}. \tag{58}$$

The above discussion can be modified to the case of the model (eq. 3) - with additional explanatory variables, where all its assumptions remain valid. As was the case for the Arellano-Bond first-difference estimator, if the variables $\boldsymbol{x}_{it}$ are correlated with the individual effect, then the procedure depends on the nature of these variables. When $\boldsymbol{x}_{it}$ are strictly exogenous, then the procedure is identical to that for the Arellano-Bond estimator and we do not obtain any additional conditions of moments to those indicated in (eq. 42). However, if $\boldsymbol{x}_{it}$ are predetermined or endogenous, then making the assumption of the truth of (eq. 52), which for the additional explanatory variables in the model (eq. 3) takes the following modified form:

$$\mathbb{E}c_i\Delta\boldsymbol{x}_{it} = 0 \text{ dla } i = 1, \dots, N; t \geq 2, \tag{59}$$

we can obtain additional moment conditions:

$$\mathbb{E}\Delta\boldsymbol{x}_{it}\xi_{it} = 0 \text{ dla } i = 1, \dots, N; t \geq 3. \tag{60}$$

It follows that lagged first differences of predetermined or endogenous variables can be instruments for the equation in levels. Finally, the system of moments conditions equation consists of the constraints defined in (44) and (60). The form of the system estimator of the generalised Blundell-Bond method of moments for the autoregressive model is analogous to that in (40), while for the model with additional explanatory variables $\boldsymbol{x}_{it}$, it is analogous to the estimator (48). However, one should remember to adopt a suitably modified weight matrix $\boldsymbol{Z}_i$. Furthermore, let us note that, analogous to the Arellano-Bond estimator, for the Blundell-Bond estimator also its one- and two-step versions are specified. Note, however, that for the purposes of the one-step Blundell-Bond estimator, the matrix $\boldsymbol{H}$ will take the following adapted form:

$$\boldsymbol{H}^{BB} = \begin{bmatrix} \boldsymbol{H} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & 1 & \dots & 0 \\ \boldsymbol{0} & 0 & \ddots & 0 \\ \boldsymbol{0} & \dots & 0 & 1 \end{bmatrix}, \tag{61}$$

that is, for the elements associated with the equation on differences, a block is left in the form of the original matrix $\boldsymbol{H}$, while the elements associated with the equation on differences correspond to a block that is an identity matrix.

To summarise the discussion of the estimation method in question, it should be mentioned that Blundell and Bond indicated that the newly proposed conditions of moments provide additional information compared to the conditions used for the Arellano-Bond estimator. Moreover, usually, even in the case of a high parameter ρ or a significant value of the ratio of the variance of the individual effect to the variance of the purely random component, the Blundell-Bond estimator is more efficient compared to the previously described methods. In this sense, the Blundell and Bond idea is developmental compared to previously proposed estimators.

2.1.4. Suboptimal system estimator of the generalized method of moments

As mentioned earlier, the use of a two-step estimation method (in the case of Arellano-Bond and Blundell-Bond estimators) is intended to improve the efficiency of the estimators. In principle, however, it is not necessarily true that as efficiency increases, the bias on the estimator will not increase. The conclusions of the Hayakawa study[76] even postulate an inverse relationship (for a large number

[76] K. Hayakawa, *Small Sample Bias Properties of the System GMM Estimator in Dynamic Panel Data Models*, Hi-Stat Discussion Paper Series, 2005, No. 82, pp. 1-28.

of instruments). In practice, in particular, it may happen that the bias on the estimator of the one-step method is lower than the bias on the estimator of the two-step method. However, due to the use of an iterative procedure in the *GMM*, the initial weight matrix is replaced by its compatible and asymptotically efficient estimator of the form (41), so the estimator of the two-step method will always be no less efficient than the estimator of the one-step method. This observation applies to both the Arellano-Bond and Blundell-Bond estimator. However, it should be noted that Bond et al.[77] show that the increase in efficiency for the two-step estimator is rather slow (slow convergence to an asymptotic distribution that is efficient). Furthermore, for two-step estimators, the standard error estimates, especially for small samples, can be significantly down-weighted. A discussion of this problem with a proposed solution is presented in *Subchapter 2.1.5.*

In view of the potential for higher biased for two-step estimators and the problems of correctly estimating standard errors, it is suggested that perhaps a one-step version of the estimators would be preferable. However, one fundamental difficulty is identified here. Namely, the optimal form of the weight matrix for one-step estimators is unknown. Two situations are an exception here. For the Arellano-Bond estimator, this is the case where the purely random error of the model is homoskedastic. Then the optimal weight matrix takes the form (38). For the Blundell-Bond estimator, the optimal weight matrix for the one-step method can only be given if the variance of the individual effect is zero. According to the work of Windmeijer[78], to obtain the optimal weight matrix in this case, the matrix $\boldsymbol{H}$ in the formula (38) has to be replaced a matrix of the following form:

$$\boldsymbol{H}_{opt}^{BB} = \begin{pmatrix} \boldsymbol{H} & \boldsymbol{A}_1 \\ \boldsymbol{A}_2 & \boldsymbol{I} \end{pmatrix}, \tag{62}$$

[77] S. Bond, A. Hoeffler, J. Temple, *GMM Estimation of Empirical Growth Models*, Centre for Economic Policy Research Discussion Paper Series, 2001, No. 3048, pp. 1-43.

[78] F. Windmeijer, *Efficiency Comparisons for a System GMM Estimator in Dynamic Panel Data Models*, IFS Working Papers, 1998, No. W98/01, pp. 1-12.

where $\boldsymbol{H}$ is given by (eq. (39), $\boldsymbol{I}$ is an identity matrix of the order $T-2$, while $\boldsymbol{A}_i$ is a quadratic matrix of the order $T-2$, having singularities on its principal diagonal, the value -1 below the principal diagonal in the case of the matrix $\boldsymbol{A}_1$ (or above the principal diagonal for the matrix $\boldsymbol{A}_2$), and zeroes elsewhere.

The question is whether it is possible to identify such an initial weight matrix for the one-step Blundell-Bond estimator that the properties (biased and efficiency) of the newly created estimator will be better than for the Blundell-Bond estimator, despite the non-zero variance of the individual effect.

Jung and Kwon[79] have undertaken a consideration of this issue. They define $\nu = \frac{\sigma_c^2}{\sigma_\varepsilon^2}$ (where σ_c^2 denotes the variance of the individual effect, while σ_ε^2 denotes the variance of the purely random error) and propose to replace the identity matrix in the formula (62) by a $\boldsymbol{J}$ matrix of the following form:

$$\boldsymbol{J} = \begin{bmatrix} 1+\nu & \nu & \dots & \nu \\ \nu & 1+\nu & \dots & \nu \\ \vdots & \vdots & \ddots & \vdots \\ \nu & \nu & \dots & 1+\nu \end{bmatrix}, \tag{63}$$

so the modified matrix $\boldsymbol{H}_{opt}^{BB}$, referred to by the authors as suboptimal, is given by:

$$\boldsymbol{H}_{sub-opt}^{BB} = \begin{pmatrix} \boldsymbol{H} & \boldsymbol{A}_1 \\ \boldsymbol{A}_2 & \boldsymbol{J} \end{pmatrix}. \tag{64}$$

Jung and Kwon show that the Blundell-Bond estimator has good properties as long as the value of ν is low. Otherwise, they show that by using the matrix $\boldsymbol{H}_{sub-opt}^{BB}$ better estimator properties will be obtained. More precisely, using Kantorovich's inequality, Jung and Kwon show that, for the proposed estimator, when ν is high, an increase in efficiency can be obtained compared to the standard Blundell-Bond estimator (both one-step and two-step)[80]. Furthermore, using

[79] H. Jung, H. U. Kwon, *An Alternative System GMM Estimator in Dynamic Panel Models*, Hi-Stat Discussion Paper Series, 2007, No. 217, pp. 1-15.

[80] For small values of ν, the increase in estimator efficiency also occurs, but its magnitude is much smaller.

Monte Carlo simulations, the researchers obtain the result that, especially in small samples, their proposed estimator (named the suboptimal systemic estimator of the generalised method of moments) is on average less biased than the Blundell-Bond estimator.

Note that in practice the ratio of the variance of the individual effect to the variance of the purely random effect ν is not known *ex-ante*. Therefore, researchers propose to replace in the $\boldsymbol{J}$ matrix the valueν by its approximation $\hat{\nu}$, defined as $\hat{\nu} = \frac{\hat{\sigma}_c^2}{\hat{\sigma}_\varepsilon^2}$, where:

$$\hat{\sigma}_c^2 = \frac{\sum_{i=1}^{N}\left(\tilde{\boldsymbol{\varepsilon}}_i{}^T\tilde{\boldsymbol{\varepsilon}}_i - \frac{\Delta\tilde{\boldsymbol{\varepsilon}}_i{}^T\Delta\tilde{\boldsymbol{\varepsilon}}_i}{2}\right)}{N(T-2)} \tag{65}$$

and

$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^{N}\Delta\breve{\boldsymbol{\varepsilon}}_i{}^{\mathrm{T}}\Delta\breve{\boldsymbol{\varepsilon}}_i}{N(T-2)}, \tag{66}$$

while $\tilde{\boldsymbol{\varepsilon}}_i$ and $\breve{\boldsymbol{\varepsilon}}_i$ are respectively the vectors of residuals from the equation on levels and the equation on differences, formed after estimating the model using the one-step Blundell-Bond method.

The approach presented by Jung and Kwon is an alternative to instrument selection using the Sargan test, which is done to reduce the bias on the estimator with a small decrease in efficiency (see *Subchapter 2.2.2.*). The suboptimal system estimator of the generalised method of moments will be used in the empirical part of this monograph presented in *Chapter 3.*

2.1.5. Adjusted variance estimator Windmeijer

Arellano and Bond[81], in addition to proposing a first-difference estimator, considered the question of the biased of the variance estimator for this method.

[81] M. Arellano, S. Bond, op. cit.

Well, as the authors conclude, the variance estimator for the one-step Arellano-Bond estimation method is unburdened, but for the two-step method it is already characterised by a significant burden, especially for small samples. Let us note that these conclusions and the following reasoning are correct for all estimators of dynamic panel models based on the generalised method of moments.

The asymptotic variance for the one-step and two-step estimator of the generalised method of moments are given by the following formulae, respectively:

$$\sigma^2(\widehat{\boldsymbol{\theta}}_1) = \frac{1}{N}(\boldsymbol{P}^T\boldsymbol{W}\boldsymbol{P})^{-1}\boldsymbol{P}^T \cdot \boldsymbol{W} \cdot \boldsymbol{W}^{-1}(\boldsymbol{\theta}_1) \cdot \boldsymbol{W} \cdot \boldsymbol{P}(\boldsymbol{P}^T\boldsymbol{W}\boldsymbol{P})^{-1}, \quad (67)$$

$$\sigma^2(\widehat{\boldsymbol{\theta}}_2) = \frac{1}{N}\left(\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\boldsymbol{P}\right)^{-1}, \quad (68)$$

where $\widehat{\boldsymbol{\theta}}_1$ is the conformal estimator of the one-step method, $\boldsymbol{W}(\boldsymbol{\theta}_1)$ is the weight matrix given by the formula (38), while $\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)$ represents the weight matrix for the two-step method given by the formula (41) . Furthermore, $\boldsymbol{P} = \frac{\partial \overline{\boldsymbol{g}}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^T}$, where $\overline{\boldsymbol{g}}(\boldsymbol{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{g}_i(\boldsymbol{\theta})$, while $\boldsymbol{g}_i(\boldsymbol{\theta})$ is the vector that occurs in the conditions of moments: $\mathbb{E}\boldsymbol{g}(\boldsymbol{x}_i\boldsymbol{\theta}) = \mathbb{E}\boldsymbol{g}_i(\boldsymbol{\theta}) = \boldsymbol{0}.$

Note that for one-step estimators, the weight matrix used does not depend on the parameters being estimated. On the other hand, for two-step estimation methods, the weight matrix (eq. (41) depends on the residuals obtained from the estimation of the one-step method. Consequently, it depends on the consistent estimator of the structural parameters of the model obtained from the one-step method.

Windmeijer[82] has shown that the bias on the two-step estimator, especially in the case of small samples, is caused precisely by the presence of the structural parameter estimators $\left[\hat{\rho}\ \widehat{\boldsymbol{\beta}}^T\right]^T$ derived from the one-step estimation, in the weight matrix used in the two-step estimation procedure. Furthermore, the author has shown that it is possible to determine the value by which the variance estimator

[82] Windmeijer F., *A Finite Sample Correction for the Variance of Linear Efficient two-step GMM Estimators*, Journal of Econometrics 126, 2005, pp. 25-51.

should be corrected for the identified biased. The researcher therefore proposes a bias-adjusted variance estimator, for the two-step estimation procedure, of the form:

$$\begin{aligned}\sigma_c^2(\widehat{\boldsymbol{\theta}}_2) = {} & \frac{1}{N}\left(\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\boldsymbol{P}\right)^{-1} + \boldsymbol{G}_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}\sigma^2(\widehat{\boldsymbol{\theta}}_2)\boldsymbol{G}^T_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)} \\ & + \frac{1}{N}\Big(\boldsymbol{G}_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}\left(\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\boldsymbol{P}\right)^{-1} \\ & + \left(\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\boldsymbol{P}\right)^{-1}\boldsymbol{G}^T_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}\Big),\end{aligned} \tag{69}$$

where $\boldsymbol{\theta} = [\rho\ \boldsymbol{\beta}^{\mathrm{T}}]^T$, and $\sigma^2(\widehat{\boldsymbol{\theta}}_1)$ is the one-step variance estimator of the *GMM* method. Furthermore, $\boldsymbol{G}_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}$ is a k^{th} degree matrix whose j^{th} column $\boldsymbol{G}_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}(j)$ is given by:

$$\begin{aligned}& \boldsymbol{G}_{\widehat{\boldsymbol{\theta}}_2,\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)}(j) \\ & = -\left(\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\boldsymbol{P}\right)^{-1}\boldsymbol{P}^T\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\left.\frac{\partial \boldsymbol{W}^{-1}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_j}\right|_{\widehat{\boldsymbol{\theta}}_1}\boldsymbol{W}(\widehat{\boldsymbol{\theta}}_1)\overline{\boldsymbol{g}}(\widehat{\boldsymbol{\theta}}_2),\end{aligned} \tag{70}$$

where :

$$\frac{\partial \boldsymbol{W}^{-1}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_j} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\partial \boldsymbol{g}_i(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_j}\boldsymbol{g}_i(\boldsymbol{\theta})^T + \boldsymbol{g}_i(\boldsymbol{\theta})\frac{\partial \boldsymbol{g}_i(\boldsymbol{\theta})^T}{\partial \boldsymbol{\theta}_j}\right). \tag{71}$$

The Windmeijer correction is usually applied to the two-step Arellano-Bond and Blundell-Bond estimators, due to the fact that these estimators use an equation on increments in the estimation process. The versions of the estimation for which the adjusted Windmeijer variance estimator is used are customarily referred to as robust (e.g. the robust two-step Arellano-Bond estimator). In the empirical part of this monograph, in *Chapter 3*, it is the robust approach that is used with the two-step estimators.

2.1.6. Other popular methods for estimating dynamic models on panel data

The authors of the studies discussed in *Subchapter 1.2.2.* use several other approaches in addition to the estimation methods outlined above. Three of them deserve to be singled out because they are not proposed for the purposes of a specific article, but have somehow entered the canon of methods for estimating dynamic models on panel data. These are: *Long-difference Instrumental Variables Estimator*, *Dynamic Panel Fractional Estimator* and *Least Square Dummy Variable Corrected Estimator*[83]. Let us note that these methods will be discussed in a cursory manner due to the fact that they were not applied in the empirical part of the paper for reasons indicated later in this subchapter.

The first estimator was originally proposed by Hahn et al.[84]. The authors estimated the following equation:

$$(\mathrm{y_{it}} - \mathrm{y_{it-k}}) = \rho(y_{it-k} - y_{it-k-1}) + \sum_{\mathrm{k}} \beta_k (x_{kit-1} - x_{kit-k-1}) \\ + (\varepsilon_{it} - \varepsilon_{it-k}), \tag{72}$$

indicating that y_{it-k-1} are good instruments for it. The parameter k represents the highest possible lag, which must be equal for all variables, hence the authors restricted themselves to the case of balanced panels only. Using the instruments indicated, the equation (72) is estimated using the two-step least squares method. The researchers indicate that the residuals from the resulting model are also correct instruments. Therefore, it is possible to re-estimate the model (72) with *the two-step least squares method* using y_{it-k-1} and the residuals obtained in the previous step as instruments. Usually, performing the above iterative procedure three times yields the final estimates (invariant over successive iterations). This approach has been extended by Huang and Ritter[85] to the case of unbalanced panels and any

[83] The names of the listed estimators, due to their lesser popularity, have no equivalent in Polish, hence English names will be used in this paper.

[84] J. Hahn, J. Hausman, G. Kuersteiner, *Long Difference Instrumental Variables Estimation for Dynamic Panel Models with Fixed Effects*, Journal of Econometrics, 2007, Vol. 140, pp. 574-617.

[85] R. Huang, J. R. Ritter, op. cit.

$k > 1$, but smaller than the minimum number of panel waves held in the dataset for any unit. In the empirical literature, the most commonly adopted approach is $k = 4$.

For corporate finance research, however, the estimator in question is not the best option, as the estimates obtained with it for many variables are statistically insignificant. Such results are obtained, for example, by Dang et al.[86] and Liang and Kebin[87]. It is also difficult to obtain an economic interpretation for this approach, because the given financial characteristics of a company do not depend significantly on the values of these characteristics several years ago[88]. In addition, the estimator in question has a very high variance for ρ close to 1[89].For the above reasons, *the Long-difference Instrumental Variables Estimator* will not be used in the empirical part of this paper.

The second estimator highlighted is the *Dynamic Panel Fractional Estimator*, which is a concept by Elsas and Florysiak[90]. This method takes into account the fact that the dependent variable is censored on both sides (in corporate finance research this is usually the interval$[0,1]$) and its distribution is saturated at zero. In this sense, the proposed method can be better matched to the nature of the dependent variable, which is very important when studying, for example, the capital structure of a company, as the distribution of the dependent variable is highly saturated at zero. *The Dynamic Panel Fractional Estimator* also takes into account the presence of an individual effect and can be used for unbalanced panels. Unfortunately, the estimation method itself is based on *Maximum Likelihood* and does not address the problem of endogeneity, and thus in the case of endogenous independent variables (a common occurrence in corporate finance), this estimator will not be consistent. As there are many endogenous variables involved in the issue of the cash holdings of listed companies, it was decided not to use the estimator in question in the empirical part of this paper.

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86 V. A. Dnag, M. Kim, Y. Shin, op. cit.

87 C. Liang, D. Kebin, op. cit.

88 In the case of having a dataset with an annual interval. For a quarterly interval, the use of the estimator in question is economically justified.

89 The problem is sometimes even identified at .$\rho = 0{,}6$

90 R. Elsas, D. Florysiak, op. cit.

The last estimator highlighted is the *Least Square Dummy Variable Corrected Estimator*. Its idea comes from the work of Kiviet[91]. It consists in determining the correction by which the fixed effects estimator for the model (eq. 3) should be corrected in order to reduce its biased in the case of small samples. The author indicates on the basis of his tests that his proposed method is better than the Arellano-Bond estimator in terms of biased. Unfortunately, a limitation of the estimator in question is the assumption of a balanced panel (very unrealistic for corporate finance research) and a set of explanatory variables consisting only of strictly exogenous variables. The first problem has been solved by Bruno[92], but the second inconvenience still remains. For this reason, as with the previously discussed method, the *Least Square Dummy Variable Corrected Estimator* will not be used in the empirical part of this paper.

Summarising the content of the above subchapter, it should be pointed out that the most popular methods of estimating dynamic models on panel data are presented, with particular emphasis on estimators based on the generalised method of moments. The historical development path of these methods is also presented. For estimators that do not use the idea of *GMM*, arguments are discussed to show why the method is not appropriate for corporate finance problems. Another very important and integral aspect of modelling is the diagnostics of the models presented. This is the subject of the next subchapter.

2.2. Model diagnostics

Methods for estimating dynamic models on panel data based on the generalised method of moments have one major advantage, namely that they are not subject to restrictive assumptions about the distributions of the individual parameters or the assumption of homoskedasticity of the random component. However, if there is a second-order correlation in the first differences of the random component in the equation on the increments - the estimator loses consistency. It is also important to check the validity of the redundant conditions

[91] J. F. Kiviet, *On Bias, Inconsistency, and Efficiency of Various Estimators in Dynamic Panel Data Models*, Journal of Econometrics, 1995, Vol. 68, pp. 53-78.

[92] G. S. F. Bruno, *Approximating the Bias of the LSDV Estimator for Dynamic Unbalanced Panel Data Models*, Economics Letters, 2005, Vol. 87, pp. 261-366.

of moments not used in the estimation process. The purpose of this subchapter is to discuss the statistical tests used to verify the aforementioned assumptions in the diagnostic process of dynamic models estimated on panel data based on *GMM*[93].

2.2.1. Arellano-Bond test

Considering the equation on differences for the model (eq. 3), which takes the form:

$$y_{it} - y_{it-1} = \rho(y_{it-1} - y_{it-2}) + \left(\boldsymbol{x}_{it}^T - \boldsymbol{x}_{it-1}^T\right)\boldsymbol{\beta} + (\varepsilon_{it} - \varepsilon_{it-1}), \quad (73)$$

It is intuitive that in the case of independence ε_{it} the first differences of the purely random error $\Delta\varepsilon_{it}$ will be correlated with each other to degree one, i.e. $\mathbb{E}\big((\varepsilon_{it} - \varepsilon_{it-1})(\varepsilon_{it-1} - \varepsilon_{it-2})\big) \neq 0$. In contrast, when we would observe a correlation in $\Delta\varepsilon_{it}$ of order higher than the first order, this would imply that the instruments used in the estimation process for the equation on differences are not appropriate. Consequently, this would imply that the estimator is not consistent.

Therefore, in order to check the consistency of the obtained estimator, Arellano and Bond[94] propose a test based on the observation that the consistency of the estimator depends on the truth of the following condition (which states that there is no second-order autocorrelation in $\Delta\varepsilon_{it}$):

$$\mathbb{E}(\Delta\varepsilon_{it}\Delta\varepsilon_{it-2}) = 0 \quad (74)$$

The null hypothesis of the test is that there is no second-order autocorrelation in $\Delta\varepsilon_{it}$, while the form of the test statistic is as follows:

$$m_2 = \frac{\Delta\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}}_{-2}\Delta\hat{\boldsymbol{\varepsilon}}_{\#}}{\sqrt{\Delta\hat{\boldsymbol{\varepsilon}}}} \sim N(0,1), \quad (75)$$

[93] Only this group of models was used for the empirical study presented in *Chapter 3*.
[94] M. Arellano, S. Bond, op. cit.

where $\Delta\hat{\boldsymbol{\varepsilon}}_{-2}$ is the vector of second differences $\Delta\hat{\boldsymbol{\varepsilon}}$, while $\Delta\hat{\boldsymbol{\varepsilon}}_{\#}$, is a vector identical to $\Delta\hat{\boldsymbol{\varepsilon}}$, but with the first two elements omitted, so that the dimensions of the vectors in the formula for the statistic m_2 allow multiplication to be performed. Proof of the fact that the statistic m_2 has a standard normal distribution is also presented in the paper by Arellano and Bond[95]. $\Delta\varepsilon_{it}$, note, moreover, that there may be a situation where neither first-order nor second-order correlation is present in (73) (for example, in the case of random straying). However, this does not detract from the validity of using the Arellano-Bond test.

Let us note that the application of the above test is justified for all dynamic models on panel data estimated by the generalised method of moments, where there is an equation on differences. For the purposes of this paper, these are the Arellano-Bond, Blundell-Bond (both two-step and one-step) and the suboptimal system estimator of the generalised method of moments. A limitation of the test in question is that it can only be applied when the number of panel waves is greater than 5. Otherwise, it is not possible to determine the vector of second differences $\Delta\hat{\boldsymbol{\varepsilon}}$, needed to determine m_2.

2.2.2. Sargan test

For the generalised method of moments, estimates of the individual parameters are obtained directly from the conditions imposed on the moments. It is therefore very important to verify their veracity, i.e. that they are uncorrelated with the random components of the model. The correctness of the conditions of moments implies the correctness of the instruments used in the estimation.

The basic test in this respect is the Sargan test, which verifies the truth of supra-identifying conditions imposed on moments and not used in the estimation process. It was developed by Arellano and Bond[96] based on a concept taken from Sargan[97] and Hansen[98] (hence other names for the Sargan test are also encountered in the literature, namely the Sargan-Hansen test or the Hansen test).

[95] ibidem, pp. 293-394.
[96] ibid.
[97] J. D. Sargan, *The Estimation of Economic Relationships using Instrumental Variables*, Econometrica, 1958, Vol. 26, pp. 393-415.
[98] L. P. Hansen, *Large Sample Properties of Generalized Method of Moments Estimators*, Econometrica, 1982, Vol. 50, pp. 1029-1054.

The test in question verifies the null hypothesis that the instrumental variables used in the estimation process are correct in the sense of being uncorrelated with purely random error. The test statistic takes the form:

$$s = \Delta\hat{\boldsymbol{\varepsilon}}^T \boldsymbol{Z} \left(\sum_{i=1}^{N} \boldsymbol{Z}_i^T \Delta\hat{\boldsymbol{\varepsilon}} \Delta\hat{\boldsymbol{\varepsilon}}^{\mathrm{T}} \mathbf{Z}_{\mathrm{i}} \right)^{-1} \boldsymbol{Z}^T \Delta\hat{\boldsymbol{\varepsilon}} \sim \chi_q^2, \tag{76}$$

where $\boldsymbol{Z}_i$ is the instrument matrix corresponding to the equation on differences, while $\boldsymbol{Z} = [\boldsymbol{Z}_1^T, \dots, \boldsymbol{Z}_N^T]^T$. The statistic has a distribution χ_q^2, where the number of degrees of freedom q, is the number of instruments minus the number of estimated model parameters.

Let us note that the use of the above test is justified for all dynamic models on panel data, estimated using the generalised method of moments, for which the equation on differences is used, as this test verifies the conditions of moments only for this equation. Additionally, in econometric practice, the Sargan test is used to limit the number of instruments used. More precisely, the instruments for variables that are predetermined and endogenous for a given t are $\boldsymbol{x}_{i1}^T, \dots, \boldsymbol{x}_{iT-1}^T$. In the estimation process, it is not necessary to use all of them (very large order of the instrument matrix) and their selection is made in the context of the lowest value of the Sargan test statistic, which is equivalent to the lowest proportion of rejections of the null hypothesis of the test. With this approach, a significant decrease in the bias on the estimator (which results from the use of a large number of instruments) can be achieved, with a small decrease in its efficiency. Furthermore, with a limited number of instruments, the computational process is simplified considerably.

A characteristic of the Sargan test is that it can only be applied when the number of instruments used in the estimation process is greater than the number of unknown parameters. In addition, when the value of the Sargan statistic is to be determined based on the residuals from estimation using one-step estimators, this is only possible if the distribution is identical and the purely random error is independent for all i and t. Otherwise, the s statistic would not have a consistent distribution χ_q^2. Therefore, in most empirical studies and statistical packages, the reported value of the Sargan test statistic for one-step estimators is the value of the same statistic for the corresponding two-step methods[99]. This approach will also be used in this paper - in the empirical study presented in *Chapter 3*. Furthermore, the Sargan test also has the disadvantage of being heavily biased against the null hypothesis when the purely random error is not homoskedastic. In addition, a significant biased of the test in favour of the null hypothesis is identified in very small samples[100].

2.2.3. Sargan differential test

The Sargan test discussed in *Subchapter 2.2.2.* was proposed with a view to checking the validity of the supra-identifying conditions imposed on the moments for the Arellano-Bond estimator. It is therefore related to the conditions of moments for the equation on differences. In the case of the Blundell-Bond estimator, additional conditions specified in (60), whose uncorrelation with the random component of the model also needs to be verified, are attached to the estimation process. For this purpose, the *Differenced Sargan Test* is proposed.

The idea of the test in question is based, as it were, on a comparison of two models - with and without constraints. The former is the systemic estimator of the generalised Blundell-Bond method of moments or the suboptimal systemic *GMM* estimator (with additional constraints imposed on the moments), while the unconstrained model is understood as the Arellano-Bond estimator (without additional conditions of moments). The null hypothesis is verified that the instrumental variables resulting from the additional conditions of moments for the equation in levels (conditions of the form (60)) are correct in the sense that they are uncorrelated with the random component of the model. The test statistic is of the following form:

$$ds = s_{OGR} - s_{AB2} \sim \chi_p^2, \tag{77}$$

[99] D. Roodman, *How to do xtabond2: An Introduction to Difference and System GMM in Stata*, The Stata Journal, 2009, Vol. 9, pp. 86-136.

[100] C. Bowsher, *On Testing Overidentifying Restrictions in Dynamic Panel Data Models*, Economics Letters, 2002, Vol. 77, pp. 211-220.

where s_{OGR} and s_{AB2} denote the values of the Sargan test s statistic for the Blundell-Bond model (or the suboptimal system *GMM* estimator) and the Arellano-Bond model, respectively, both obtained from the two-step method. In the absence of grounds to reject the null hypothesis, the *ds* statistic has a distribution χ_r^2, where the number of degrees of freedom is equal to the number of additional moment conditions for the Blundell-Bond estimator, compared to the moment conditions used in the Arellano-Bond estimation process. Let us note that the use of the differential Sargan test is also justified in the case of the suboptimal system *GMM* estimator, discussed in *Subchapter 2.1.4.*

In summary, the above subchapter discusses three basic tests used in the diagnostic process of dynamic model estimation on panel data, the idea of which is based on *GMM* - the Arellano-Bond test, the Sargan test and the differential Sargan test. They allow verification of the basic assumptions of these models, namely the assumption of no second-order correlation in the first differences of the random component and the assumption of correctness of the instruments used. All the tests discussed above were used in an empirical study, the results of which are presented in *Chapter 3.*

To summarise the content presented in *Chapter 2*, it should be pointed out that various methods of estimating dynamic models on panel data have been discussed in a way that makes it possible to see the validity of the evolution of econometric methods in this area. A summary of the most important characteristics of the discussed estimation methods is presented in *Table A. 2.* in the Appendix. It shows that the estimators presented have a different spectrum of applications, mainly depending on the characteristics of the dataset held. It is impossible to clearly identify a universal method. In the case of corporate finance research, where there are numerous endogenous variables and significant heterogeneity of entities, methods based on the generalised method of moments seem to be the most appropriate. However, it is impossible to know in advance which method from this group will be the best in terms of bias and efficiency, so it seems reasonable to verify this issue in the empirical part of this paper. In addition to presenting the development of methodologies for estimating dynamic models on panel data, *Chapter 2* also addresses the issue of their diagnostic process. The Arellano-Bond test is presented, verifying the hypothesis of the absence of second-order correlation in the first differences of the purely random error, and the Sargan test and the differential Sargan test verifying the correctness of the instruments in the sense of their uncorrelation with the random component, for the instruments used in the equation on increments (differences) and levels, respectively.

CHAPTER 3

PROPERTIES OF DYNAMIC PANEL MODEL ESTIMATORS ON THE EXAMPLE OF MODELLING THE TRANSACTIONAL LIQUIDITY RESERVE OF LISTED COMPANIES IN POLAND

The purpose of this chapter is to present an empirical study of the properties of estimators for dynamic panel models, using a real-world example of an issue in corporate finance - modelling the size of the Cash Holdings of listed companies in Poland. This will make it possible to verify the research hypotheses posed in the introduction of the paper.

The chapter be gins with a description of the database and the variables used in the study. The variables defined previously were then used to run *Monte Carlo* simulations under four different simulation scenarios. First, the effect of the size of the true parameterρ on the properties of its estimators was considered, followed by a discussion of the importance of the strength of the effect of the other regressors on the explanatory variable in the context of parameter estimation with a lagged dependent variable. This made it possible to verify the first auxiliary *hypothesis* (*Hypothesis H1)* that the lack of variation in the strength of the influence of the individual explanatory variables on the dependent variable may result in a reduced burden and improved precision of the parameter estimates ρ. An analysis of the estimators considered was then presented, varying the number of waves of the panel used for modelling. This made it possible to verify the second auxiliary hypothesis (*Hypothesis H2*) that the length of the panel adopted for the study determines the choice of an adequate estimation method. The auxiliary hypothesis, that the spectrum of possible estimation methods for dynamic models on panel data is significantly narrowed when there is a correlation between the individual effect and the initial values of the explanatory variable (*Hypothesis H3*), was verified by the fourth scenario simulation, which discusses the effect of the distribution of the individual effect and the purely random component on the properties of the parameter estimators ρ.

The discussion is concluded with a summary of the results obtained in the framework of the simulations carried out, which lead to the final verification of the main hypothesis of this paper (*Hypothesis MH*), stating that despite continuous improvements in the methodology of estimating dynamic models on panel data, the best estimation method for empirical research in corporate finance based on this type of models cannot be unambiguously indicated. However, it is possible to identify indications that, in some cases, indicate the most appropriate estimation method for the issue under consideration.

3.1. Description of the database and variables

Polish empirical literature emphasises the liquidity aspects of enterprises mainly in the context of their liquidity as a whole, i.e. the ability to make purchases and settle liabilities in full and on the applicable dates[1]. Moreover, Polish researchers devote their analyses mainly to entities in the SME sector and, in addition, do not generally decompose liquidity levels into transactional and additional liquidity reserves. The main interest of this paper is not the problem of modelling the size of the Cash Holdings, but the issue of comparing econometric methods of estimating the parameter indicating the speed of adjustments of the size under study, however, an empirical study of the properties of estimators was decided to be carried out on the basis of real data and a specific economic issue. Therefore, it was decided to adopt for the empirical study of the properties of estimators for dynamic panel models the issue of Cash Holdings of listed companies in Poland. Thanks to this choice, in addition to considering econometric issues, it was possible to at least partially fill the gap in the Polish literature described above. The purpose of this subchapter

[1] U. Wojciechowska, *Płynność finansowa polskich przedsiębiorstw w okresie transformacji gospodarki. Aspekty mikroekonomiczne i makroekonomiczne*, Warsaw: Oficyna Wydawnicza SGH 2001, ISBN 83-7225-098-7, pp. 14.

is to describe the database used in the empirical part of the study and to characterise the variables defined on its basis used in the subsequent econometric modelling.

3.1.1. Description of the database

The dataset used in the empirical part of the study was created by collecting information from two different sources. They were mostly taken from the *NOTORIA Poland* website. In particular, the *NOTORIA Poland* database includes annual financial statements of listed companies listed on the Warsaw Stock Exchange, which is the primary source of data for the empirical part of this study. More precisely, the basic dataset was created on the basis of financial data of all companies listed on the Main Market and the *NewConnect* Market in 1999-2012. In addition, information on the consumer price index (based on data published by the Statistical Office[2]) was also added to the dataset created in this way. The purpose of such a procedure is the subsequent calculation of the real values of the company's balance sheet total for the purpose of characterisation reflecting the size of the company. Let us further note that the variables used for modelling have been carefully defined and discussed in *Subchapter 3.1.2.*

The study focuses exclusively on non-financial entities, which, as defined by the Central Statistical Office[3], determines the exclusion from the research sample of companies be longing, according to the Polish Classification of Activities 2007, to sections K (Financial and insurance activities), A (Agriculture, forestry, hunting and fishing) and O (Public administration and defense, compulsory social security). The exclusion of the first group is dictated by the different nature of assets and liabilities compared to the other companies. Banks are identified in the context of capital with the supply side, while the other companies represent the demand side. In addition, both insurance institutions and banks are subject to regulations requiring them to maintain liquid assets at an appropriate level[4]. Sections A and O, on the other hand,

[2] http://stat.gov.pl/obszary-tematyczne/ceny-handel/wskazniki-cen/wskazniki-cen-towarow-i-uslug-konsumpcyjnych-pot-inflacja-/roczne-wskazniki-cen-towarow-i-uslug-konsumpcyjnych-w-latach-1950-2014, Accessed 23 April 2015.

[3] Non-financial business activity in 2010, On line, http://stat.gov.pl/cps/rde/xbcr/gus/pgwf_dzialalnosc_przedsiebiorstw_niefinansowych_w_2010.pdf

[4] E.g. the Solvency II Directive or the Basel Committee on Banking Supervision regulations.

are characterised by significantly different business characteristics from the other groups of companies. Consequently, there is no rationale for their Cash Holdings to be analysed together with the cash of other entities. By narrowing the area of interest to non-financial companies only, it was possible to obtain a research sample with greater homogeneity in terms of the company's motivation to maintain a Cash Holdings, which does not result from top-down regulations and restrictions. It was also decided to exclude from the research sample companies characterised by a very weak financial situation, where it can be assumed that the entity is on the verge of bankruptcy. These companies reported a ratio of total liabilities to total assets greater than unity, a negative value of equity or provided financial statements in spite of prior deletion from the National Court Register (Polish Company Register) (*e.g.* companies reporting in the course of ongoing liquidation proceedings). In order to reduce the impact on the analysis of outliers, a winsorisation of all continuous variables used for modelling was performed. This consists of replacing values smaller than the first percentile and greater than the ninetieth percentile of their distributions by the values of these percentiles. An analogous procedure was carried out in the empirical studies discussed in *Subchapter 1.2.*

The final dataset is an unbalanced panel, containing information on 642 entities. The total number of observations amounts to 3688. Let us note that due to the lack of data in the case of individual variables, some companies listed on the Warsaw Stock Exchange were not included in the research sample. However, the final set includes data on 75% of entities from the group under consideration (taking into account the exclusion of companies classified in sections A, K and O according to NACE 2007). In this sense, the sample adopted for the study can undoubtedly be considered representative.

In summary, based on information from the financial statements of companies listed on the Warsaw Stock Exchange (sourced from the *NOTORIA Poland* service) and additional data on the consumer price index sourced from the Statistical Office portal, a final dataset was created for the empirical part of this study. On its basis, the variables used for modelling were defined. Their detailed description along with the analysis of descriptive statistics is presented in the next subchapter.

3.1.2. Characteristics of the variables used

From the point of view of econometric modelling, a very important aspect is the appropriate selection and definition of variables. By doing so, it is possible to negate the problem of omitted variables, which can result in biased on the estimators. Thus, if the influence of one of the primary sources of biased is minimised, it will be possible to make a more reliable comparison of properties between the different estimation methods (biased due to omitted variables could slightly distort these results).

The choice of definition of the explanatory variable is undoubtedly an important issue. As mentioned in the introduction to this subchapter, Polish researchers use different definitions of explanatory variables in the context of liquidity studies than foreign researchers. They most often use the current liquidity ratio, the accelerated liquidity ratio and the cash ratio, in line with the definitions of the Statistical Office[5]. However, wishing to keep the approach comparable with the empirical studies presented in *Subchapter 1.2.,* it was decided to adopt the definition of the explanatory variable in accordance with the definition of cash at the end of the period, within the meaning of Resolution No. 5/11 of the Accounting Standards Committee of 10.05.2011 on the adoption of the revised national accounting standard No. 1 “Cash flow statement”[6].

According to the above document, cash at the end of the period includes cash and cash equivalents. Cash and cash equivalents are monetary assets in the form of domestic means of payment, securities with a monetary function (foreign exchange) and foreign currencies held in cash or bank accounts. Cash equivalents, on the other hand, are other assets not classified as cash, which are characterised by a low risk of impairment, a short maturity and, above all, a high degree of liquidity, i.e. a relatively simple and inexpensive conversion into cash. In addition, for the purpose of determining the value of the explanatory variable, the value of cash at the end of the period was weighted by the size of the company's assets. This allowed comparability between specific entities and within a single company over

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[5] M. Sierpińska, T. Jachna, *Assessing enterprises according to world standards*, Warsaw: PWN 2014, ISBN 978-83-01-14987-1, pp. 145-149.

[6] Official Gazette of the Minister of Finance No. 6 of 2 August 2011.

time. Using the value of total assets as a divisor seems quite intuitive, in that it is the simplest approximation of company size. Some authors (such as Opler et al.[7]) apply a logarithmic transformation to the aforementioned variable in order to overcome the problem of its limitation. However, due to the low yield of this approach[8] and the desire to maintain comparability to most empirical studies, it was finally decided to adopt the definition of the explanatory variable as the quotient of cash at the end of the period and total assets, which is consistent with the definition of value as defined in the English-language literature by *Cash Holdings*. The formal definitions of all variables used in the study are included in *Table 2*.

The survey in question was carried out using panel data and therefore the values of the individual descriptive statistics given for the entire survey period taken together may not be overly informative. Therefore, the values of the mean, median, minimum, maximum and both quartiles for the continuous variables used in the study are presented as graphs in *Figure 2*. In addition, due to the relevance of the explanatory variable to the study as a whole, the exact values of its descriptive statistics together with the size of the individual panel waves are provided in *Table A.1.* Let us note that the imbalance of the panel (see in *Table A.1.*). is mainly due to the entry of new companies on the stock exchange and the emergence of the *NewConnect* market.

As this paper focuses on the econometric issues related to the estimation of dynamic models on panel data, the values of the descriptive statistics for the individual continuous variables used in the study will not be analysed within the framework of this paper in terms of their economic interpretation (however, the motivation for the selection of the individual variables for the study will be presented). Let us only note that *the Cash Holdings*[9] is characterised by a mean

[7] T. Opler, L. Pinkowitz, R. Stulz, R. Williamson, *The Determinants and Implications of Cash Holdings*, Journal of Financial Economics 52, 1999, pp. 3-46.

[8] An analysis (not reported in the paper) of the econometric models considered was carried out for the explanatory variable defined as $\ln\left(\frac{\text{Cash and cash equivalents}}{\text{Total Assets}}\right)$, but no significant improvement in the fit between the theoretical values of the model and the distribution of empirical values of the explanatory variable was obtained.

[9] If this does not lead to confusion, the explanatory variable will be referred to as the cash holdings or by equivalent names, i.e. cash or most liquid assets.

value similar to that obtained from the empirical study presented in *Subchapter 1.2.*, with a lower value at the beginning of the period under study and a higher value between 2007 and 2009. This may illustrate the maintenance of a higher liquidity reserve by listed companies, in response to the global financial crisis and the consequent increased probability of bankruptcy of their global counterparties.

Table 2. Summary definitions of the variables used in the study.

Variable	Definition of the variable
EXPLANATORY VARIABLE	
Cash Holdings	$\frac{\text{Cash and cash equivalents}}{\text{Total Assets}}$
EXPLANATORY VARIABLES	
Cash Holdings at *t-1*	$\left(\frac{\text{Cash and cash equivalents}}{\text{Total Assets}}\right)_{t-1}$
Company size	$\ln(\text{Total Assets})$
Self-financing	$\frac{\text{Operating cash flow}}{\text{Total Assets}}$
Debt ratio	$\frac{\text{Total liabilities}}{\text{Total Assets}}$
Debt ratio²	$\left(\frac{\text{Total liabilities}}{\text{Total Assets}}\right)^2$
Funding deficit	$\frac{\text{capital expenditure} + \text{dividends paid} - \\ - \text{cash flow}}{\text{Total Assets}}$
Maturity matching	$\frac{\text{Long} - \text{term liabilities}}{\text{Total liabilities}}$
Tax rate	$\frac{\text{Tax paid income}}{\text{Gross profit}}$
Net working capital	$\frac{\text{Short} - \text{term Assets} - \text{Short} - \text{term liabilities}}{\text{Total Assets}}$
Business development opportunities	$\frac{Sales_{\mathrm{t}} - \mathrm{Sales}_{\mathrm{t}-1}}{Sales_{\mathrm{t}-1}}$
Investment expenditure	$\frac{\mathrm{ppe}_{\mathrm{t}} - ppe_{\mathrm{t}-1} + \\ + \mathrm{amortisation}_{\mathrm{t}}}{ppe_{\mathrm{t}-1}}$
Return on assets (ROA)	$\frac{\text{Net profit}}{\text{Total Assets}}$
Payment of dividends	binary variable (1-firm pays dividends, 0-other)

Source: own study.

The motivation for the inclusion of each variable in the set of regressors derives directly from an interest in those characteristics of the various economic theories concerning the Cash Holdings, which are briefly discussed in *Subchapter 1.1.1.* A detailed economic analysis of each variable, in the context of the individual theories, is presented in Mirota and Nehrebecka examining the determinants of the size of the Cash Holdings held by listed companies[10]. Only a brief description of the relevance of the use of the variable in question as a regressor will be provided within the framework of this discussion.

Figure 2. Graphs of basic descriptive statistics for the continuous variables used in the study.

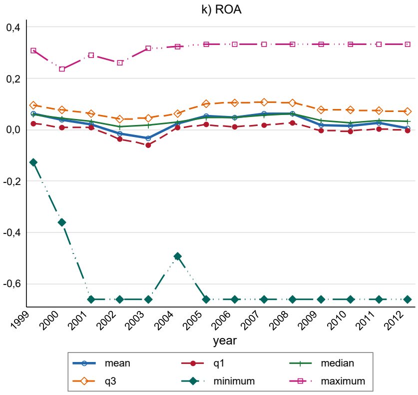
A Continuation of *Figure 2* can be found on the next page.

[10] Mirota F., Nehrebecka N. (2018), Determinants of cash holdings in listed companies in Poland, National Economy, no. 3, pp. 75-102.

Continued *Fugure 2*

Continuation of *Figure 2* can be found on the next page.

Continued *Fugure 2*



Source: own compilation based on *NOTORIA Poland* data.

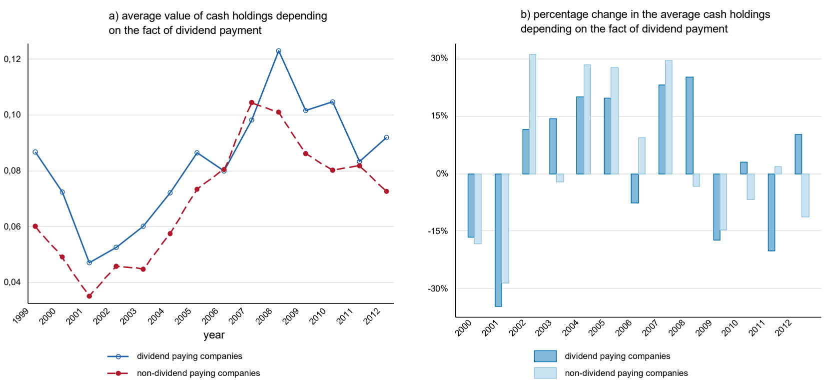
One of the most important characteristics of a company that speaks of the scale of its operations is its *Size*, usually understood as the value of the company's total assets. The logarithm of this size is taken as the explanatory variable (in line with the trend prevailing in the literature), but after converting total assets into real terms by means of the CPI. Another issue that it is reasonable to consider is the possibility that the company's operations can be *self-financed* from its current operating cash flow (cash flow as equivalent to *the Cash Holdings* will have a significant impact on its level). However, it is very often the case that a company relies on external financing, which generates significant liabilities. An indicator of the size of these liabilities is called *the Debt Ratio* and will also be used in this study. Another measure reflecting the degree to which a company is financially self-sufficient is the *Funding Deficit*, understood as the difference between capital expenditure and dividends paid and the size of cash flow (scaled by the size of the company's assets). Companies can, of course, take a number of measures to control their debt. One of these is *Maturity Matching* (understood as the ratio of long-term liabilities to total liabilities), which can make it significantly easier for a company to make the required payments (they will be matched to the flows the company receives). Another way to settle liabilities faster is to increase the *Return on Assets*, so that any external financing yields, on average, higher returns per unit of borrowed capital. All of the above-mentioned characteristics (indicated *in italics* as the names of the variables) are directly related to the level of *the Cash Holdings*, as they determine the way in which the company's activities are financed and affect issues related to its liquidity (of which the Transactional Reserve is a component).

The next two variables introduced in the study are related to the company's growth. In order for this to be possible, the company should incur *Expenditure on investment*, which, according to the theory of the hierarchy of sources of financing, will be financed first from own funds, thus reducing the stock of *Cash Holdings*. On the other hand, companies with greater *Growth Opportunities* will, in the light of the free cash flow theory, maintain a higher stock of cash (because, from the point of view of the company's owners, it will be more difficult to detect investments abandoned in order to achieve their own objectives by its managers, so they will be able to afford to accumulate the most liquid assets more easily).

From the point of view of the shareholders of a listed company, the fact that it declares *a Dividend Payment* is very important. Interestingly, the impact of such a declaration on the size of the cash holdings can be twofold. On the one hand, retaining part of the profit earmarked for dividend payments may increase the size of cash (in this sense, we observe a positive relationship between the declaration of dividend payments and the size of the company's most liquid assets). However, on the other hand, if the shareholders are also the managers of the firm, they will want to pay dividends even at the expense of reducing the firm's cash holdings (in this sense, we expect an inverse relationship). *Figure 3* shows the average (and dynamics) of *the Cash Holdings* depending on the fact that the firm pays dividends. It shows that, as a rule, dividend-paying companies maintain, on average, a higher level of cash than companies that do not make distributions to their shareholders (consistent with the theory of the hierarchy of sources of financing).

In addition, due to the fact that the *Cash Holdings* is a component of *Working Capital*, there is a fairly strong correlation between the two variables (Spearman correlation coefficient of 0.5192), hence it is reasonable to use the mentioned variable in the modelling. The Spearman correlation matrix for all continuous variables used in the study is presented in *Table A.3*. It shows that the correlation between the individual regressors is low, so we do not have the problem of collinearity that can reduce the precision of the estimates. The last variable used is the *Tax Rate*, which is understood as a real quantity (tax paid income to gross profit). According to Bigella and Sanchez-Vidal[11] companies subject to a higher tax rate will maintain a lower balance of their most liquid assets, because the higher the taxation, the higher the opportunity cost of holding cash.

Figure 3. Graph for the variable *Cash Holdings* depending on the fact of dividend payment.



Source: own compilation based on *NOTORIA Poland* data.

Due to the limitations of the available database, other characteristics considered in the literature were not included in the analysis (in particular, variables relating to the ownership and management structure of the firm, which are significantly emphasised by Ozkan and Ozkan[12]). However, this should not fundamentally affect the conclusions of a study focusing on the econometric properties of estimators for dynamic panel models.

In summary, the purpose of the above subchapter was to describe the database from which the dependent variable was determined and the set of regressors was selected and characterised. Let us note that although the main idea of *Chapter 3* is to present the results of an empirical study of the properties of the estimators (in terms of the parameter ρ) used to estimate dynamic models

[11] M. Bigelli, J. Sanchez-Vidal, op. cit.
[12] A. Ozkan, N. Ozkan, op. cit.

on panel data, it is impossible to omit the discussion of the variables used in the *Monte Carlo* simulation, which the above subchapter accomplishes. This serves as a sort of introduction to the content presented in the following subchapters.

3.2. Basic assumptions on the simulations carried out

The purpose of this subchapter is to present the general assumptions about the simulation procedure that are common to all simulation scenarios. Furthermore, the idea of the *Monte Carlo* method, which is used to study the properties of the parameter estimators ρ for different estimation methods and different experimental designs, is briefly discussed. The discussion concludes with a presentation of preliminary estimates of the model (eq.3) made using five estimation methods, based on the variables defined in the previous chapter.

As mentioned earlier, the very idea of the study is based on running *Monte Carlo* simulations to compare the properties of the obtained estimators of the parameter ρ. Due to the fact that, when examining the general properties of the estimators, one expects results at a higher level of generality than just for the data set used, it was decided to draw up four separate simulation scenarios varying in turn: the true magnitude of the parameter ρ, the true magnitudes of the coefficients β_k, the number of panel waves, and the distributions of individual effect and purely random error. The considerations carried out in this respect finally lead to conclusions allowing to verify the hypotheses posed in the introduction of the paper.

The model of the form (eq. 3), on which the estimations were carried out, is distinguished by several characteristics. Namely, there is a lagged dependent variable in the set of its explanatory variables, the individual entities may be characterised by significant heterogeneity, and some of the regressors may be endogenous. Accordingly, adequate estimators (from the range presented in *Chapter 2*) for estimating this type of model are the Arellano-Bond estimator (both one- and two-step), the Blundell-Bond estimator (both one- and two-step) and the suboptimal systemic generalised method of moments estimator. The rationale for the selection of these methods (and the rejection of the others) is presented in *Chapter 2*. The simulations were finalised based on the group of five estimation methods listed above.

The simulation procedure itself uses the standard procedure of the *Monte Carlo* method for studying the properties of estimators. In general, it relies on the assumption that we know some data generating process that depends on the parameter vector $\boldsymbol{\theta}$. The objects of interest in this case are the expected value and the variance of the estimator $\hat{\rho}$, which can be formally written as:

$$\mathbb{E}(\hat{\rho}) = F_1(\boldsymbol{\theta}, N) \equiv \phi_1, \tag{78}$$

$$Var(\hat{\rho}) = \mathbb{E}(\hat{\rho} - \phi_1)^2 = F_2(\boldsymbol{\theta}, N) \equiv \phi_2, \tag{79}$$

where $F(\cdot)$ is some function depending on the sample and parameter vector $\boldsymbol{\theta}$. Based on *a Monte Carlo* simulation with M iterations, the values of ϕ_1 and ϕ_2 can be estimated as:

$$\bar{\phi}_1 = \frac{1}{M}\sum_{i=1}^{M} \hat{\rho}_i, \tag{80}$$

$$\bar{\phi}_2 = \frac{1}{M}\sum_{i=1}^{M} (\hat{\rho}_i - \bar{\phi}_1)^2. \tag{81}$$

In practice, for the study carried out, the data generating process is based on the equation (eq. 3). For the basic version of the simulation (also referred to as the baseline), the variables $\boldsymbol{x}_i$ were assumed to be based on the dataset at hand, while the parameters ρ and β_k were assumed to be equal in magnitude to the estimates for the individual models as estimated on the entire dataset. More precisely, this means that initially the model (eq. 3) was estimated by five different estimation methods, based on the research sample in possession. It was then assumed that, for the purposes of the *Monte Carlo* simulation, the parameter vector $\boldsymbol{\theta} = [\hat{\rho}, \widehat{\boldsymbol{\beta}}]$, Where $\hat{\rho}$ and $\widehat{\boldsymbol{\beta}}$ are the estimates of the individual parameters for a given estimation method. Further y_{it} values are generated based on the equation (eq. 3) with the parameters appropriately assumed, with $c_i \sim U[-1,1]$ and $\varepsilon_{it} \sim N(0,1)$ assumed as in the Flannery and Hankins study[13]. This is a fairly intuitive and commonly used approach, because according to it, a firm's individual effect can have both positive and negative effects on the explanatory variable with equally distributed probability on a symmetric interval. The random error, on the other hand, is derived from a standard normal distribution. In addition, the initial value of y_{it} in each uninterrupted sequence of observations for a given subject was taken according to the available dataset.

Note that the data generation process was carried out separately for each estimation method due to differences in the size of the parameters $\hat{\rho}$ and $\hat{\beta}_k$. A model of the form was re-estimated on the new data sets prepared in this way, knowing the true values of the parameters $\rho_{new} = \hat{\rho}$ and $\beta_{k,new} = \hat{\beta}_k$ for them:

$$y_{it}^{sym} = \rho_{new} y_{it-1}^{sym} + \sum_{\mathrm{k}} \beta_{k,new} x_{kit} + c_i + \varepsilon_{it}, \tag{82}$$

with the individual estimation methods. Estimates $\hat{\rho}_{new}$ were thus obtained for each method separately. The above procedure was repeated 500 times. Thanks to the *Monte Carlo* simulation carried out in this way, it was possible to subsequently determine the biased of the estimator, its empirical variance and the prediction errors of the parameter ρ_{new}.

Note that the above-described method of performing *Monte Carlo* simulations is presented using the baseline scenario as an example, while the other simulation scenarios are based on varying assumptions about the size of the parameters $\rho_{new}, \beta_{k,new}$ the number of *T-panel* waves and the distribution of c_i and ε_{it}. These will be presented in detail before discussing the results for the individual experiments. Furthermore, the parameters in the *Monte Carlo* simulation are denoted by the subscript$nowy$, while the inference itself will, of course, proceed for the parameter ρ (following the model designations (eq.3)).

Two advantages of this simulation approach should be mentioned in the context of the studies presented so far in the literature comparing the properties of estimators for dynamic panel models. Firstly, the whole *Monte Carlo* procedure is based as much as possible on real data. This made it possible to keep the structure

[13] M. J. Flannery, K. W. Hankins, op. cit., pp. 7.

of the simulation set consistent with the structure of the database for real observations. In the simulations presented in the empirical studies presented in *Subchapter 1.2.2.*, the authors usually generate the explained variables $\boldsymbol{x}_{it}$ according to an *AR(1)* (or similar) process, which can significantly distort the structure of the real data. Secondly, all of the studies presented in *Subchapter 1.2.2.*, if they use a database with real company characteristics, it is the *Compustat* database. However, each dataset has its own differences in terms of methodological definitions of variables as well as statistical properties. In particular, as noted by Dang, Kim and Shin[14] dynamic panel models estimated on many different subsamples of the *Compustat* database are characterised by the presence of second-order correlations in first differences of purely random errors. This may be crucial for the selection of adequate estimation methods, as estimators based on the generalised method of moments may be characterised by inconsistency in this case. Finally, let us note that the use of the same database by the authors of the study (even if they select different subsamples from it) in some way gives the results they obtain an airtight character. Consequently, an undoubted added value of the present study is the use of a unique dataset for the simulation. In addition, the size of the cash holdings was taken as the economic problem in the context of which the properties of the estimators for dynamic panel models are considered. To date, the authors have conducted similar research (discussed in *Subchapter 1.2.2.*) based on the issue of firm capital structure (not including the article by Liang and Kebin[15], which was prepared for an academic conference and is not published within any regular journal, moreover, it still contains some gaps). In this sense, this paper is at least a partial filling of the indicated gap in the literature.

Due to the relevance to the simulation process of the standard estimates of the model (eq. 3) made on the basis of the available database by means of the Arellano-Bond estimator (both one-step and two-step), the Blundell-Bond estimator (both one-step and two-step) and the suboptimal system *GMM* estimator, the results of the mentioned standard estimates will be briefly discussed before presenting the results of the *Monte Carlo* simulations themselves. The results of these estimations are included in *Table 3*, with the coefficients

[14] V. A. Dang, M. Kim, Y. Shin, op. cit.
[15] C. Liang, D. Kebin, op. cit.

relating to the binary variables for the individual years from which the observations were taken (a time effect that was included in the model) not included for presentational convenience.

In all models, *Self-financing* and *Net Working Capital* are assumed as endogenous variables (except for the lagged dependent variable). For both of them, the simultaneity causing the endogeneity problem is identified. Indeed, the *Self-Financing* variable depends on cash flows, which directly affect *the Cash Holdings* (as presented in *Figure 2.*). Moreover, as mentioned earlier, cash is part of net working capital. Relationships in the opposite direction are assumed according to the form of the estimated equation. The remaining variables are assumed to be exogenous variables. This approach to determining the nature of the variables determines the adoption as instruments for the equations at the lagged first differences levels of *Cash Holdings, Self-Financing* and *Net Working Capital*. Due to the small number of instruments, it was possible to strike a balance between the improvement in estimation efficiency (resulting from the additional instruments) and the potentially possible increase in burden (if too many instruments were adopted). A detailed list of the instruments used in the estimation process (both for the equation on increments and on levels) is included under *Table 3*. In addition, for the two-step estimators, the adjusted Windmeijer variance estimator was used. Note that the estimation of the model (82) in the *Monte Carlo* simulation for all scenarios used assumptions identical to those discussed above about the nature of the variables and the instruments adopted and the approach to using the adjusted Windmeijer variance estimator.

No objections were noted to the estimated models, from an econometric point of view. On the basis of the Arellano-Bond test, there are no grounds for rejecting the null hypothesis of no second-order correlation in the first differences of purely random error for any of the models. Furthermore, there are also no grounds for rejecting the null hypothesis of the validity of the instruments used in the sense of their not being correlated with the purely random component, both for the equation on increments (based on the Sargan test conducted for all models considered) and for the equation on levels (based on the differential Sargan test conducted for the one- and two-step Blundell-Bond estimator and the suboptimal system *GMM* estimator).

As the economic interpretation of the models obtained is not related to the purpose of this paper, let us only note that the obtained directions of the influence of individual explanatory variables on the cash holdings clearly indicate the superiority of the postulates of the theory of hierarchy of sources of financing over the theory of substitution in explaining the variability of the explained variable. Thus, listed companies in Poland prefer to finance their operations first from their own funds, then from debt, and at the very end from share issues[16]. Furthermore, let us note that *the half-life* of the size of the most liquid assets to the intended level, after a unit shock in the random component, is relatively short. Depending on the model, it ranges from just over six months to just over eight months.

Estimates of the parameter with the explained variable lagged by one period already provide us with some information, but they vary between methods (the main difference is visible between methods using the equation for levels and those that do not use this equation). Let us note, however, that the true value of the parameter ρ remains unknown all the time, and its estimates between different methods differ significantly (e.g. in the context of interpretation of the half-time of adjustment). This makes it all the more pertinent to study the properties of the various estimators for this parameter, in order to be able to indicate the exact scope of their applicability and to offer guidelines useful to researchers carrying out empirical studies in corporate finance.

In summary, the above subchapter presented the basic idea of *Monte Carlo* simulations used in relation to the study of the properties of estimators. This is followed by a presentation of the basic assumptions for the simulations performed in this monograph to help verify the research hypotheses. The subchapter concludes with a presentation of the results of the estimates of the considered model (eq. 3) by means of five selected *GMM-based* estimation methods, which estimates are crucial from the point of view of the data-generating process parameters adopted for the subsequent simulations.

[16] Wider economic considerations on this topic are presented in F. Mirota and N. Nehrebecka, op. cit.

Table 3. Estimation results of dynamic cash holdings models on panel data using methods based on the generalised method of moments.

Variable	**One-step Arellano-Bond first difference estimator**		**Two-step Arellano-Bond fizArst difference estimator**		**One-step Blundell-Bond system *GMM* estimator**		**Two-step Blundell-Bond system *GMM* estimator**		**Suboptimal system GMM estimator**	
	coefficient	[statistic; p-value].	coefficient	[statistic; p-value].	coefficient	[statistic; p-value].	coefficient	[statistic; p-value].	coefficient	[statistic; p-value].
Cash Holdings at *t-1*	0.2987***	[6.91; 0.000]	0.2971***	[6.64; 0.000]	0.3389***	[8.15; 0.000]	0.3404***	[7.85; 0.000]	0.3342***	[8.12; 0.000]
Company size	0.0375***	[5.34; 0.000]	0.0390***	[5.44; 0.000]	0.0221***	[6.63; 0.000]	0.0225***	[6.34; 0.000]	0.0242***	[6.89; 0.000]
Self-financing	0.1679***	[7.13; 0.000]	0.1472***	[6.21; 0.000]	0.1420***	[5.73; 0.000]	0.1364***	[5.66; 0.000]	0.1447***	[5.67; 0.000]
Debt ratio	-0.1493**	[-2.47; 0.013]	-0.1591***	[-2.73; 0.006]	-0.2448***	[-4.59; 0.000]	-0.2436***	[-4.65; 0.000]	-0.2483***	[-4.29; 0.000]
Debt ratio^2	0.1904***	[3.03; 0.002]	0.1726***	[2.78; 0.005]	0.2279***	[3.91; 0.000]	0.2181***	[3.68; 0.000]	0.2436***	[3.99; 0.000]
Funding deficit	-0.2299***	[-7.58; 0.000]	-0.2486***	[-7.70; 0.000]	-0.2504***	[-8.30; 0.000]	-0.2568***	[-7.63; 0.000]	-0.2555***	[-8.44; 0.000]
Maturity matching	-0.0854***	[-4.97; 0.000]	-0.0711***	[-4.30; 0.000]	-0.0657***	[-3.90; 0.000]	-0.0626***	[-3.82; 0.000]	-0.0639***	[-3.91; 0.000]
Tax rate	-0.0063*	[-1.75; 0.081]	-0.0052	[-1.48; 0.138]	-0.0067*	[-1.82; 0.069]	-0.0072**	[-1.97; 0.049]	-0.0063***	[-1.75; 0.081]
Net working capital	0.3042***	[7.88; 0.000]	0.2687***	[6.82; 0.000]	0.2302***	[7.31; 0.000]	0.2280***	[6.69; 0.000]	0.2401***	[7.75; 0.000]
Business development opportunities	-0.0052*	[-1.68; 0.093]	-0.0050	[-1.55; 0.122]	-0.0046	[-1.56; 0.119]	-0.0051*	[-1.78; 0.075]	-0.0040***	[-1.36; 0.174]
Investment expenditure	-0.0022*	[-1.85; 0.064]	-0.0023*	[-1.89; 0.059]	-0.0018	[-1.48; 0.140]	-0.0017	[-1.41; 0.159]	-0.0020***	[-1.77; 0.076]
Return on assets (ROA)	-0.0754***	[-4.15; 0.000]	-0.0718***	[-4.18; 0.000]	-0.0702***	[-3.94; 0.000]	-0.0746***	[-4.13; 0.000]	-0.0719***	[-4.07; 0.000]
Payment of dividends	0.0091*	[1.83; 0.067]	0.0105**	[2.12; 0.034]	0.0118**	[2.25; 0.024]	0.0118**	[2.31; 0.021]	0.0104***	[2.02; 0.044]
factor λ	70.13%		70.29%		66.11%		65.96%		66.58%	
Half-life	0.57		0.57		0.64		0.64		0.63	
Arellano-Bond test		[-1.04; 0.301]		[-1.13; 0.259]		[-0.50; 0.614]		[-0.45; 0.654]		[-0.71; 0.477]
Sargan test		[261.63; 0.374]		[261.63; 0.374]		[287.76; 0.5426]		[287.76; 0.5426]		[299.14; 0.359]
Sargan differential test						[26.13; 0.1391]		[26.13; 0.1391]		[37.51; 0.6452]
F_{year} test		[46.24; 0.000]		[37.88; 0.000]		[34.60; 0.000]		[34.22; 0.000]		[39.19; 0.000]

For the equation on increments, the following instruments were used: cash holdings$_{t-2}$, self-financing$_{t-1}$, networking capital$_{t-1}$, Δcompany size, Δdebt ratio, Δdebt ratio^2, Δdeficit financing, Δmaturity mismatch, Δtax rate, Δcompany growth opportunities, Δcapital expenditure, ΔROA, Δdividend payment, Δyear$_{2001}$ - Δyear$_{2012}$.
For the equation on levels, the following instruments were used: Δcash holdings$_{t-1}$, Δself-financing$_{t-1}$, Δnet working capital$_{(t-1)}$.
For the two-step estimators, the adjusted Windmeijer variance estimator was used.
For all models, the time effect was taken into account by introducing binary variables for each panel wave (coefficients not reported in the table).
The symbols ***, **, * denote the statistical significance of the parameters at the 1%, 5% and 10% significance levels respectively.
Arellano-Bond test - test for the presence of second-order correlation in first differences of the random component.
Sargan test - test for the validity of the instruments in the equation on the increments, in the sense of their being uncorrelated with the random component of the model.
Differential Sargan test - a test for the correctness of the instruments in the equation in levels, in the sense of their being uncorrelated with the random component of the model.
F_{year} test - test of pooled non-significance of variables representing the time effect ($Year_j$).

Source: own compilation based on *NOTORIA Poland* data.

3.3. Simulation results and conclusions

The following subchapter presents the results of *Monte Carlo* simulations aimed at investigating the econometric properties of the estimator of the parameter standing in the equation (eq. 3) with the explanatory variable lagged by one period. The considerations were carried out in terms of four different groups of simulation scenarios, within which the assumptions about the size of the parameters $\rho_{new}, \beta_{\boldsymbol{k},new}$ (in terms of equation (82)), the number of *T-panel* waves and the distribution of c_i andε_{it} are modified (relative to the base case described in *Subchapter 3.2.)*.

As part of the presentation of the results for each group of simulations, box plots of the biased for the individual estimators of the ρ parameter and plots of the empirical variance of the ρ parameter estimates are presented. In addition, in order to assess the accuracy of the prediction of the parameter ρ by the econometric models under consideration, charts of the most popular measures of prediction error and the Theil's U statistic are presented. As the values of the parameterρ are smaller than unity, the most appropriate measures of prediction error will be relative measures - mean relative prediction error (*MAPE*) and mean adjusted prediction error (*AMAPE*). Therefore, graphs of the values of these two measures will be presented in the body of the paper when considering the results of *Monte Carlo* simulations, while graphs for the other measures (*MAE, MSE, RMSE*) and the Theil's U statistic are included in *Appendix B*. A brief description of the individual measures, together with how they are determined, is presented in *Appendix C*.

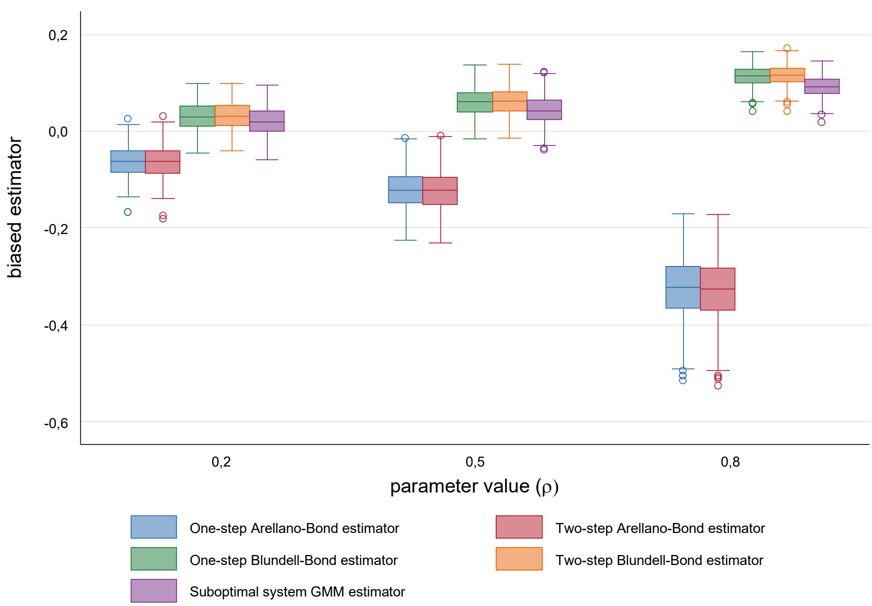
3.3.1. Effect of the size of the coefficient on the lagged dependent variable on the properties of the estimators

The first simulation group focuses on the effect of the magnitude of the true parameter ρ on the bias and variance of the estimates obtained, using the selected models. For the purpose of the *Monte Carlo* simulation, ρ_{new} was adopted from 0.1 to 0.9 with an interval of 0.1.

Figure shows box plots of the biased of the individual estimators of the parameter ρ as a function of its true value. For the sake of presentational clarity, it was decided to include only the results for $\rho_{new} \in (0.2; 0.5; 0.8)$ in the graph under consideration. Note that for the purposes of this chart (as well as all others presented in this subchapter), in order to maintain consistency of designations, the parameter ρ, which is not subject to direct economic interpretation, is discussed; nevertheless (as noted in *Subchapter 1.1.1.).* $1 - \rho = \lambda$ represents the rate of adjustment of the explained variable to the optimal level.

The results of the simulations clearly show a significant increase in the nominal biased of the considered estimators of the parameter ρ as its true value increases (especially for ρ close to unity). The main reason for this is that the problem of weak instruments, i.e. weak correlation of instrumental variables with explanatory variables, is exacerbated with increasing ρ. Overcoming this problem is one of the main ideas guiding the developers of estimators using the equation in levels. The undoubted effectiveness of its application is evident in *Figure 4*, in the much lower biased for the Blundell-Bond estimators and the suboptimal system *GMM* estimator than for the Arellano-Bond estimators.

Figure 4. Box plots of the biased estimators ρ against the true value of the parameter ρ.



Source: own compilation based on *NOTORIA Poland* data.

The average values of the estimates of the parameter ρ as a function of its true value presented in *Figure 5* a further indicate that the increase in nominal biased for the Arellano-Bond estimators (both one-step and two-step) increases with the increase in the true value of the estimated parameter in a much faster than linear manner (especially for $\rho > 0{,}6$). This can be particularly acute for authors carrying out empirical studies in corporate finance, who determine and interpret the rate of adjustment of the quantity under study to the level assumed by the company. In particular, when the Arellano-Bond estimator is used, the values of the half-time of adjustment of the explanatory variable can be heavily skewed. This is particularly true if the true value of this time is longer than for the case whose results are presented in *Table 3*. Then the value of the coefficient ρ is higher and, consequently, the biased on the Arellano-Bond estimator also takes on a larger value.

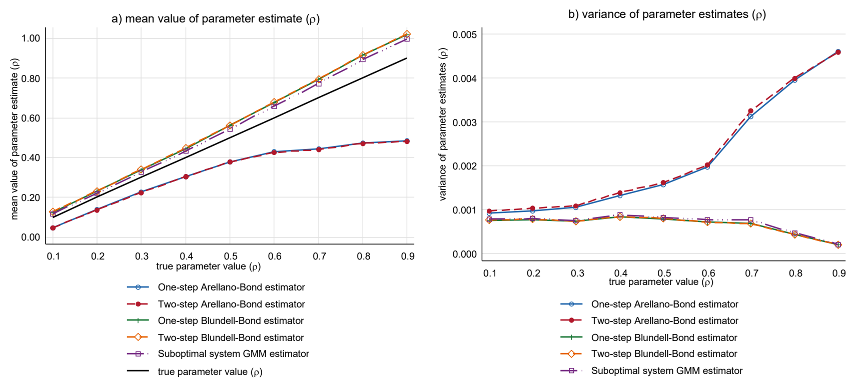
Furthermore, let us note that, as a rule, the parameter estimates ρ are down-weighted for Arellano-Bond estimators (both one-step and two-step). The occurrence of such a situation has already been pointed out by the authors of the aforementioned method themselves[117]. Furthermore, the problem of weak instruments, which increases with the true value of the parameter ρ, has a very negative impact on the precision of the Arellano-Bond estimators (this is also noted in their study by Blundell and Bond[118]). This is highlighted in *Figure 5 b.* In addition, it can be inferred that the assumed efficiency improvement for estimators using the equation in levels does indeed take place (irrespective of the use of an asymptotically efficient weight matrix).

As for the Blundell-Bond estimators and the suboptimal system *GMM* estimator, they are slightly biased upwards in the results presented. This is not a general rule, as the direction of biased depending on the database at hand can be ambiguous. It is further noteworthy that the prediction error measures shown in *Figure 6* have significantly higher values for extremely small true values of the parameter ρ. Admittedly, the nominal biased value for these cases is not significant, but with respect to the true value of the parameter ρ this can have a very significant impact on economic inference. This is particularly evident for the

[117] M. Arellano, S. Bond, op. cit.
[118] R. Blundell, S. Bond, op. cit.

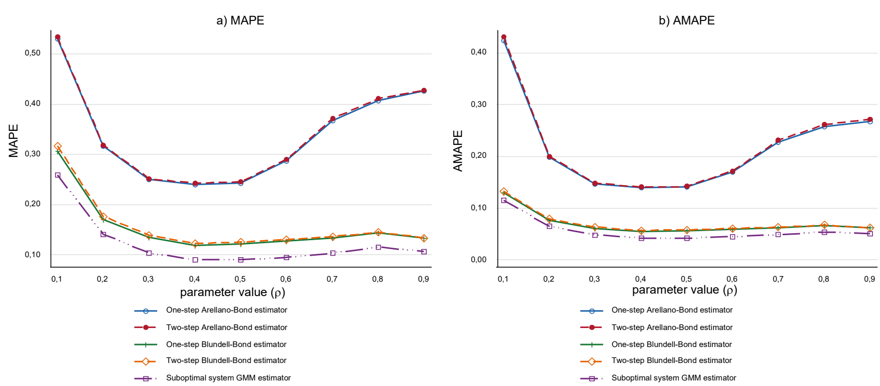
outlier observations in *Figure 5* for the Arellano-Bond estimators for $\rho = 0.2$, Where the magnitude of the downward biased of the estimator is almost equal to the true value of the estimated parameter. As can be seen from *Figure 6* in the extreme case, for the Arellano-Bond estimator and $\rho = 0.1$ the average prediction error can be more than 50% of the true value of the estimated parameter.

Figure 5. Mean value of parameter estimates ρ and value of the empirical variance of parameter estimates ρ for the different estimation methods depending on the true value of the parameter ρ.



Source: own compilation based on *NOTORIA Poland* data.

Figure 6. Relative measures of the prediction error of the parameter ρ (MAPE, AMAPE) for the different estimation methods depending on the true value of the parameter.ρ



Source: own compilation based on *NOTORIA Poland* data.

In summary, the true value of a parameter with a lagged dependent variable is an important determinant of the properties of the estimator of that parameter, which is consistent with results presented in the literature (see *Subchapter 1.2.2.*). It can be pointed out that better properties in this respect are characterised by estimation methods using the equation in levels, with the suboptimal system *GMM* estimator having a lower biased than the Blundell-Bond estimator and having only a slightly higher variance. On the basis of the Theil's U statistic (*Figure B.1 d*), it can also be indicated as the most appropriate estimation method for the problem under consideration. Let us additionally note that for the smallest true values of the parameter ρ, in spite of the small nominal bias it can be severe and have a key impact on the conclusions of the empirical study, which is generally not indicated by the authors.

3.3.2. Impact of the size of the coefficients at the other explanatory variables on the properties of the estimators

The second group of simulations annotates the consideration of the effect of the size of the true coefficients $\beta_{k,new}$ on the properties of the parameter estimators ρ. Four cases of *Monte Carlo* simulations were considered as a result of modifying the assumptions of the base model in terms of parameter sizes $\beta_{k,new}$:

- baseline scenario,
- scenario adopted $\beta_{k,new} = 0.2$ for each, $i = 1, \dots, k$,
- a scenario adopting $\beta_{k,new} = \hat{\beta}_k + 2\sigma_{\hat{\beta}_{k,new}}$, where $\hat{\beta}_k$ is an estimate of the coefficient β_k, from the model (eq.3) estimated on the whole available dataset (see *Table*), using the given estimation method. In contrast,$\sigma_{\hat{\beta}_{k,new}}$ represents the standard deviation of the estimate of $\hat{\beta}_k$,
- scenario adopted $\beta_{k,new} = \hat{\beta}_k - 2\sigma_{\hat{\beta}_{k,new}}$, where the designations are consistent with those described above.

The idea of scenario two ($\beta_{k,new} = 0.2$) originates from the work of Kiviet[119], where the author focused on the one-period lagged explanatory variable without varying the strength of influence of other explanatory variables in explaining the variation in the dependent variable. Thus, they could be introduced into the model by some aggregate. An identical approach is sometimes used by authors of articles aiming to compare the properties of estimators (e.g. Flannery and Hankins[120], or Zhou, Faff and Alpert[121]).

The idea behind the last two scenarios is respectively:

- increasing the positive direction of the effect of individual regressors on the dependent variable (or decreasing the strength of the negative effect) by adding two standard deviations of this estimate to the β_k estimate,
- increasing the negative direction of the effect of individual regressors on the dependent variable (or decreasing the strength of the positive effect) by subtracting two standard deviations of this estimate from the β_k estimate.

Figure 7 shows box plots of the biased of individual estimators depending on the assumptions made about $\beta_{k,new}$. It shows that varying the strength of the influence of the independent variables on the explanatory variable is not fundamentally reflected in the difference between the properties of the estimators of the parameterρ. Noteworthy, however, is the reduced biased for $\beta_{k,new} = 0.2$ compared to the other scenarios. Then the variables $\boldsymbol{x}_{it}$ could theoretically be introduced into the model in some aggregate. With this representation of the case, the raised variation in one of the regressors could be offset by the values of the other explanatory variables. This results, in principle, in a lower bias on the individual estimators than in the case of varying strength of influence of the independent variables on the variability of the characteristic under study. Similarly, it also has a positive effect on the size of the variance of the parameter estimates ρ (*Figure 8*).

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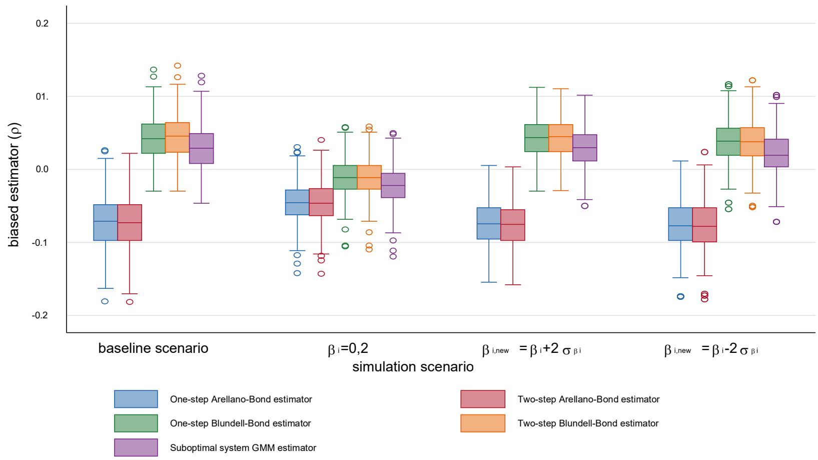
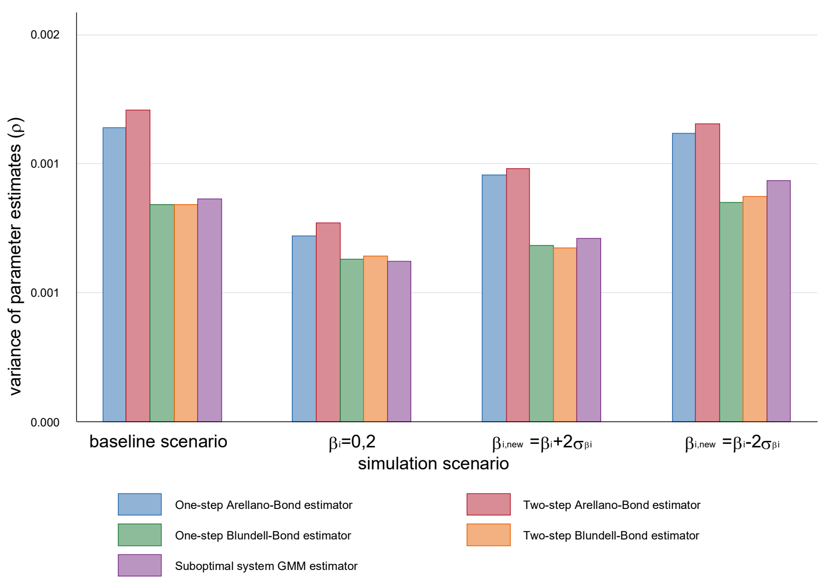
[119] J. Kiviet, op. cit.
[120] M. J. Flannery, K. W. Hankins, op. cit., pp. 7.
[121] Q. Zhou, R. Faff, K. Alpert, op. cit.

Figure 7. Box plots of the biased estimators ρ depending on the quantities adopted for the simulation $\beta_{k,new}$.



Source: own compilation based on *NOTORIA Poland* data.

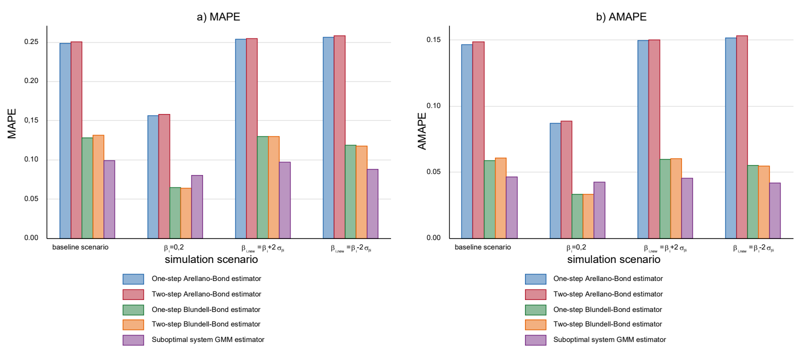
Figure 8. Value of the empirical variance of the estimates of the parameter ρ for the different estimation methods depending on the quantities adopted for the simulations $\beta_{k,new}$.



Source: own compilation based on *NOTORIA Poland* data.

Figure 9, showing the relative measures of prediction error of the parameterρ, confirms the previously cited conclusions about the improved properties of the estimators in the absence of variation in the strength of influence of the individual explanatory variables on the dependent variable. However, one interesting property concerning the suboptimal systemic *GMM* estimator should be noted. Namely, *MAPE* and *AMAPE* for this estimation method are higher than for the Blundell-Bond estimators in the case of $\beta_{k,new} = 0.2$. This is mainly due to the higher biased of the suboptimal system *GMM* estimator than the Blundell-Bond estimators, for the scenario in question. This is interesting as the modified weight matrix, using the $\boldsymbol{H}_{sub-opt}^{BB}$ matrix of the form (eq. 64) was supposed to reduce the bias on the Blundell-Bond estimator. However, Jung and Kwon[122] propose and test a suboptimal systematic *GMM* estimator based on data with varying strength of influence of the explanatory variables on the independent variable. Let us note that the increase in the aforementioned biased is very insignificant, nevertheless, the observed property could be a field for developing considerations of a suboptimal system *GMM* estimator for the case of an autoregressive model and a model with explanatory variables for which the coefficients β_k are equal.

Figure 9. Relative prediction error measures of the parameter ρ (MAPE, AMAPE) for the different estimation methods depending on the quantities adopted for the simulation $\beta_{k,new}$.



Source: own compilation based on *NOTORIA Poland* data.

[122] H. Jung, H. U. Kwon, op. cit.

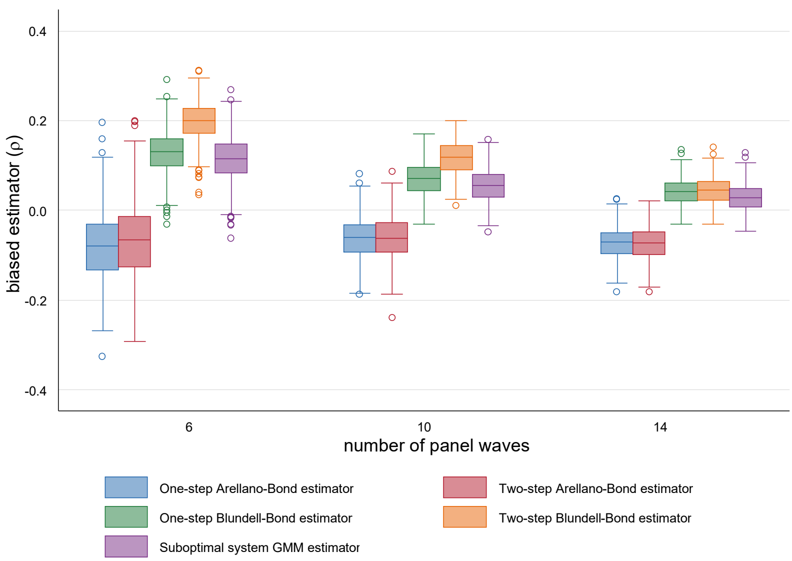
In summary, the magnitudes of the true coefficients β_k (in the sense of the model (eq. 3)) are generally not important determinants of the properties of the estimators for dynamic panel models estimated by methods based on the generalised method of moments. An exception to this may be the case when the strength of the influence of individual regressors on the dependent variable is identical. In that case, the biased on the estimators under consideration is slightly reduced. Therefore, there are no grounds for rejecting the first auxiliary *hypothesis* (*Hypothesis H1*), that the lack of variation in the strength of the influence of individual explanatory variables on the dependent variable may cause a reduction in the bias and improve the precision of parameter estimates ρ. Consequently, a treatment that may go some way to improving the properties of the estimators used in empirical corporate finance research is to scale the variables $\boldsymbol{x}_{it}$ so that the strengths of their effects on the dependent variable are similar to each other. Unfortunately, this can make the economic interpretation of the results more difficult, and the yield of such a procedure may be low in some cases (e.g. due to the difficulty of performing an appropriate rescaling associated with the biased of the parameter estimates β_k).

3.3.3. Effect of panel length on the properties of estimators

Another group of simulation scenarios involves taking panel data with different wave numbers into the *Monte Carlo* procedure. The theoretical literature (used in *Chapter 2*) indicates that the length of the panel is an important determinant of the size of the bias for the estimators of dynamic panel models, which will be verified in this subchapter on real data on listed companies in Poland. In order to modify the number of waves of the panel, the full data set was truncated to a specific wave count, with the most recent data being used for simulations, *i.e.* for example, in the case of the assumption that *T=10*, data from *2003-2012* were used for estimation. Moreover, the shortest panel tested had *6* waves.

For presentational convenience, *Figure 10* showing box plots of the biased of the parameter ρ as a function of the length of the panel used is $T \in (6, 10, 14)$. The scenario with *T=14* is the same as the baseline scenario described in *Subchapter 3.2*. In principle, the bias on all estimators decreases as the number of panel waves increases, which is in line with theory. Particularly noteworthy, however, is the high (compared to the Arellano-Bond estimators) biased of the estimators using the equation in levels, for small T. This has to do, among other things, with taking as instruments the lagged values of the first differences of the endogenous and predetermined variables. This, of course, requires a larger panel wavelet size than in the case of the equation on differences (the lagged levels of these variables, rather than their lagged first differences, are used as instruments of the non-exogenous variables).

Figure 10. Box plots of the biased estimators ρ as a function of the number of T panel waves.



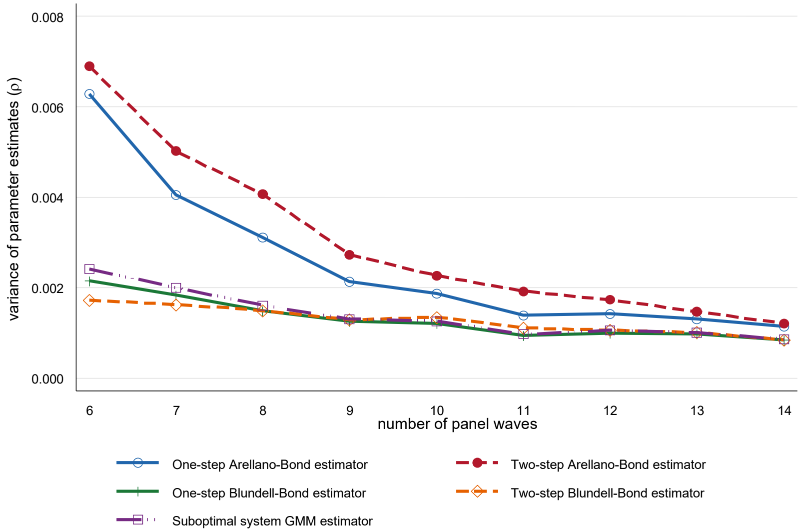
Source: own compilation based on *NOTORIA Poland* data.

In addition, as the initial weight matrix for the Blundell-Bond method was replaced by its asymptotically efficient estimator, the problem described in *Subchapter 2.1.4.* materialised significantly for $T \leq 10$. Namely, the use of an asymptotically efficient weight matrix for the two-step Blundell-Bond estimator resulted in a significant increase in its biased compared to the one-step version. Furthermore, as presented in *Figure 1*, the increase in efficiency of the two-step method is basically only noticeable for the shortest of the analysed panels, where the increase in bias on the two-step Blundell-Bond estimator is the largest.

Analysing the prediction error measures (*Figure* –12) one can see the problem of increased biased of the two-step Blundell-Bond estimator for $T \leq 10$. In summary, it can be seen from the above analysis that the length of the panel adopted for the study is crucial to the properties of the parameter estimatorρ and should determine the choice of estimation method. For the shortest panels (for the considered dataset $T < 8$), the Arellano-Bond estimators will be the most appropriate estimators in terms of biased and prediction errors, but unfortunately they will have a significant variance. For $T \geq 10$ it makes a lot of sense to consider estimators using the equation in levels. This is also confirmed by the Theil's U statistic presented in *Figure B.3.d*. However, the use of the two-step Blundell-Bond estimation procedure requires particular caution, as at the expense of increased efficiency (compared to the one-step procedure) there was a significant increase in the bias on the parameter estimator ρ. Ultimately, the more appropriate estimation methods for the (eq. 3) model for slightly longer $T > 10$ panels appear to be the Blundell-Bond one-step estimator and, above all, the suboptimal system *GMM* estimator. The above considerations indicate that there are no grounds for rejecting the second auxiliary hypothesis of this monograph (*Hypothesis H2*), that the length of the panel adopted for the study determines the choice of an adequate estimation method.

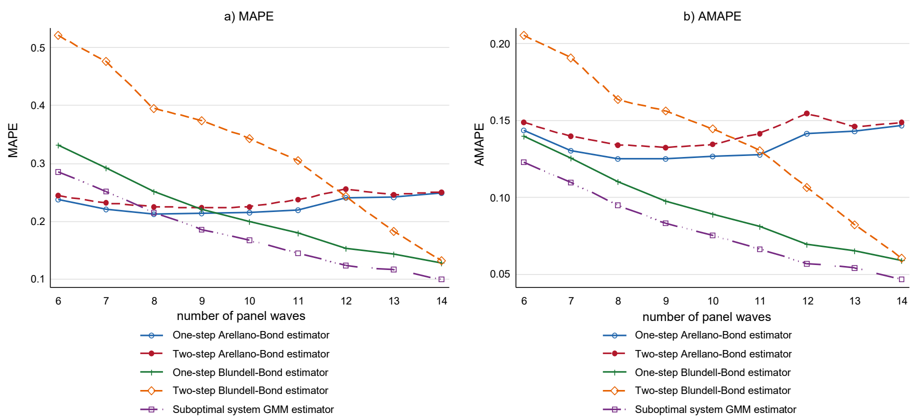
In practice, thanks to new data collection techniques, panels are characterised by an increasing number of waves, so for most corporate finance research the problem of an extremely short panel ($T < 8$) should not arise. Nevertheless, it may be the case that for countries that have only recently started to collect reliable company data, the length of the possible panel data is not very satisfactory. In this case, the economic analysis should be treated with some caution and consideration should be given to the question of testing the properties of particular estimation methods on the data set in hand.

Figure 11. The value of the empirical variance of the parameter estimates ρ for the different estimation methods as a function of the number of waves of the *T* panel.



Source: own compilation based on *NOTORIA Poland* data.

Figure –12. Relative prediction error measures of the parameter ρ (MAPE, AMAPE) for the different estimation methods as a function of the number of *T* panel waves.



Source: own compilation based on *NOTORIA Poland* data.

3.3.4. Effect of individual effect distribution and random component distribution on the properties of the estimators

The last group of simulation scenarios considered the problem of how the distribution of the individual effect c_i and the purely random error ε_{it} affect the properties of the parameter estimators ρ. This was done by varying the random distribution of both variables for the equation (82). The following four scenarios were considered for the *Monte Carlo* simulation:

- baseline scenario, in which $c_i \sim U[-1,1]$ and $\varepsilon_{it} \sim N(0,1)$ (the ratio of the variance of the individual effect to the variance of the purely random effect is $\frac{1}{3}$),
- scenario with $c_i \sim U[-0.25,0.25]$ and $\varepsilon_{it} \sim N(0,1)$ (the ratio of the variance of the individual effect to the variance of the purely random effect has been significantly reduced with respect to the baseline scenario and is $\frac{1}{48}$),
- scenario with $c_i \sim U[-0.25,0.25]$ and $\varepsilon_{it} \sim N(0,0.25)$ (the ratio of the individual effect variance to the purely random effect variance has been slightly reduced with respect to the baseline scenario and is $\frac{1}{12}$),
- a scenario that assumed $c_i \sim U[-1,1]$ and ε_{it} from a logistic distribution with mean 0 and scale parameter $s = 0.055$ and $\beta_{k,new} = 0.2$ for each k.

The last of the scenarios was based on the study of purely random errors from models pre-estimated on the entire dataset (for which estimation results are included in *Table 3*). The values of the basic characteristics of the distribution $\hat{\varepsilon}_{it}$ for the different estimation methods are included in *Table 4*. From these it can be concluded that the overall mean of the purely random error is close to zero, while the variance is close to 0.01. The distributions $\hat{\varepsilon}_{it}$ for all the estimation methods considered are slightly right-skewed and have increased kurtosis compared to the normal distribution (they are more soaring). The carrier of the distribution for a purely random error should be the set of real numbers. Consequently, in the popular range of probability distributions, it is impossible to find a distribution whose characteristics coincide with those shown in *Table 4*. One compromise that has been decided upon is to adopt for the purely random error a logistic distribution with mean 0 and scale parameter $s = 0.055$ (this corresponds to a variance equal to 0.01). This distribution has a kurtosis equal to 1.2, while it does not include right skewness (the skewness of the logistic distribution is zero).

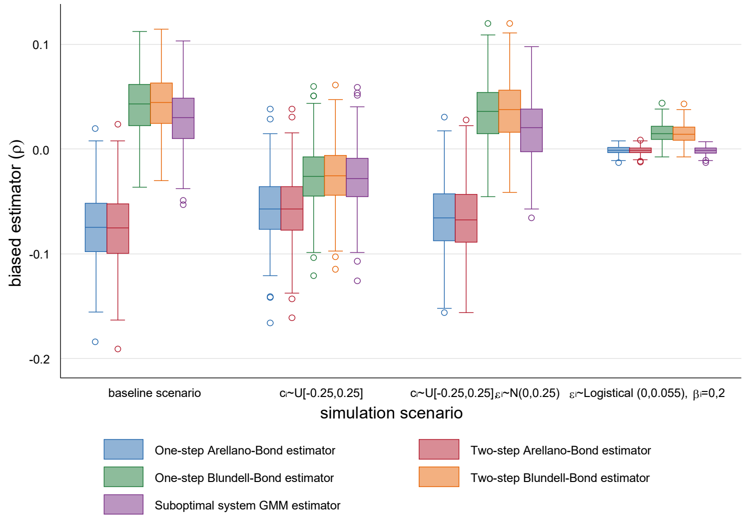
Table 4. Values of basic distribution characteristics $\hat{\varepsilon}_{it}$ from model estimates (eq. 3) on the full dataset using individual estimation methods.

Estimation method	**Average**	**Variation**	**Skewness**	**Kurtosis**
One-step Arellano-Bond estimator	- 0.0949	0.0119	0.8700	1.0979
Two-step Arellano-Bond estimator	- 0.0937	0.0119	0.8941	1.1679
One-step Blundell-Bond estimator	0.0044	0.0101	0.9472	1.2376
Two-step Blundell-Bond estimator	0.0062	0.0102	0.9492	1.2400
Suboptimal system *GMM* estimator	- 0.0197	0.0107	0.9246	1.1750

Source: own study.

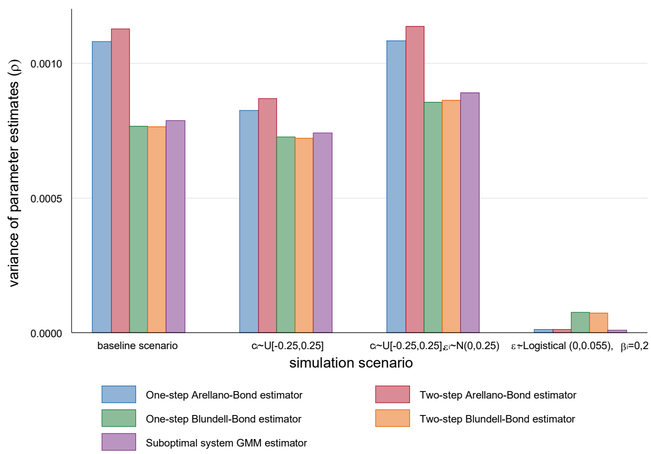
Box plots of the biased estimators ρ for the individual scenarios are presented in *Figure 13*. It can be deduced from it that for the first three simulation concepts, the biased value for all the estimators considered decreases as the value of the quotient of the variance of the individual effect and the variance of the purely random error decreases. This is related to the fact that for higher values of this quotient, the problem of weak instruments intensifies. The higher the value of this quotient is, the higher we can see the yield of the suboptimal system *GMM* estimator, which is in this respect an improvement of the Blundell-Bond estimator. Let us further note that the direction of the average biased for estimation methods using the equation in levels, changes between scenarios two and three. However, this is not a general regularity (in the case of decreasing values of the variance ratio c_i and ε_{it}), but the effect of a slight perturbation of the sample structure resulting from a change in the distribution parameters c_i and ε_{it}. The changes in the values of prediction errors for the first three scenarios in this group (*Figure* 15 and Figure B.4.) are in line with the directions of changes in biased - they are smaller the less significant the problem of weak instruments is. As for the variance of the parameter estimates ρ for the first three scenarios, within them, its absolute values for a given estimation method are basically convergent (*Figure 14*).

Figure 13. Box plots of the biased estimators ρ against the individual effect distribution and the purely random error distribution.



Source: own compilation based on *NOTORIA Poland* data.

Figure 14. The value of the empirical variance of the parameter estimates ρ for the different estimation methods depending on the distribution of the individual effect and the distribution of the purely random error.

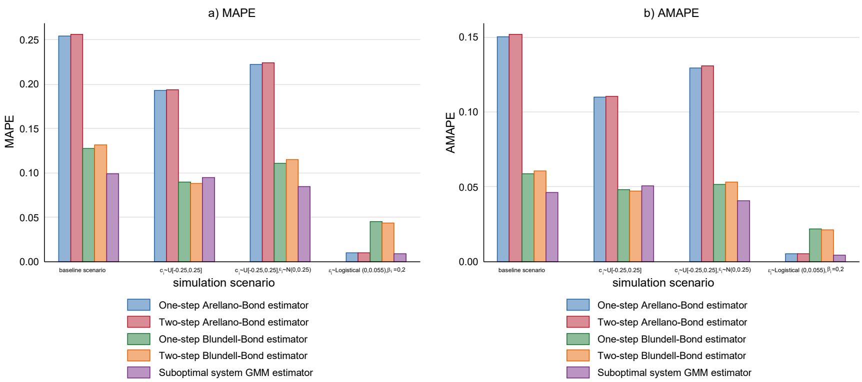


Source: own compilation based on *NOTORIA Poland* data.

The last simulation case considered within this group gives very interesting results. The motivation for assuming a purely random error from a logistic distribution has been presented above, while the motivation to $\beta_{k,new} = 0.2$ for each$k$ is quite technical, but very important. Namely, in the case of assuming $\beta_{k,new} = 0.2$, the values generated y_{it}^{sym} will be positive on average (according to the nature of the explanatory variable). If this assumption is not made, for ε_{it} with a variance equal to 0.01, the value of c_i is crucial for the sign of y_{it}^{sym} (in particular, it may be negative). Consequently, c_i significantly determines y_{it}^{sym}, and consequently also Δy_{i2}^{sym}. Consequently, there may be indications that the given problem does not satisfy the assumption (eq. 52) imposed on the initial conditions. This may cause a significant burden on the Blundell-Bond estimators and consequently a lack of justification for their applicability compared to the Arellano-Bond estimators (which are not subject to this assumption). By adopting $\beta_{k,new} = 0.2$, this problem is reduced for the situation Where such a treatment is not applied, but it is not nullified completely, as reflected by the higher biased for the Blundell-Bond estimators than for the Arellano-Bond estimators (*Figure 13*). For real data, such a situation does not generally occur and the assumption (eq. 52) is met, but nevertheless a scenario in which the Arellano-Bond estimator has an advantage (in terms of biased and efficiency) over the Blundell-Bond estimator, due to not meeting all model assumptions, is not impossible.

Furthermore, when comparing the scenario under consideration with the second scenario from the group presented in *Subchapter 3.3.2.*, lower biased values can be observed for the currently discussed group of simulations. This illustrates that the appropriate selection of the error distribution purely random to the data can have some influence on the results of the *Monte Carlo* simulations carried out. Consequently, the results of simulations adopting the normal distribution as the purely random error distribution may have slightly skewed results (higher biased, variance and prediction errors than if the purely random error distribution were selected for the data resulting from the initial model estimates). However, the inference itself remains unchanged, and the above commentary relates only to simulation comparisons of the properties of the estimators and has no bearing on the consideration of purely economic empirical studies.

Figure 15. Relative prediction error measures of the parameter ρ (MAPE, AMAPE), for individual estimation methods depending on the individual effect distribution and the purely random error distribution.



Source: own compilation based on *NOTORIA Poland* data.

In summary, when conducting an empirical study of corporate finance based on a dynamic panel model, when a very low variance$\hat{\varepsilon}_{it}$ is identified, estimation methods using the equation in levels (and especially the suboptimal systemic estimator of the generalised method of moments) should be used. Such a recommendation is based on the fact that a very low variance $\hat{\varepsilon}_{it}$ may indicate a high value of the ratio of the variances c_i andε_{it}, and consequently a problem of weak instruments. Furthermore, in the case of identifying significantly higher values of relative prediction errors for Blundell-Bond estimators than for Arellano-Bond estimators, the fulfilment of the (eq. 52) condition imposed on the initial conditions should be verified, and if it is not fulfilled, the Arellano-Bond method should be used for the final estimation and estimation methods exploring the equation in levels should not be used. It follows that there are no grounds for rejecting the third auxiliary hypothesis of this paper (*Hypothesis H3*) stating that the presence of a correlation between the subject's individual effect and the initial values of the explanatory variable significantly narrows the spectrum of possible estimation methods for dynamic models on panel data.

Summarising the contents presented throughout *Chapter 3*, it presents a comprehensive study of the properties of estimators for dynamic panel models, using the example of modelling the cash holdings of listed companies in Poland.

The initial part of the discussion presents a description of the database used and characterises all the variables used in the study. In addition, the *Monte Carlo* simulation procedure for testing the properties of estimators for dynamic panel models using the generalised method of moments is discussed. The second part of the chapter presents the results of the groups of simulations carried out, which investigated in turn the effects of: the size of the coefficient on the lagged dependent variable, the strength of the influence of the individual variables on the dependent variable, the panel length, and the distributions of the individual effect and the pure random error, on the properties of the estimators of the parameter ρ. The conclusions obtained indicate that there are no grounds to reject any of the auxiliary hypotheses of this study. Namely, there are no grounds to reject the truth of the statements that: the lack of variation in the strength of the effect of individual explanatory variables on the dependent variable can reduce the burden and improve the precision of estimates of the parameter ρ (*Hypothesis H1*), the length of the panel adopted for the study determines the choice of an adequate estimation method (*Hypothesis H2*), and that the presence of correlations between the individual effect of the subject and the initial values of the explanatory variable significantly narrows the spectrum of possible methods for estimating dynamic models on panel data (*Hypothesis H3*).

Ultimately, this leads to the conclusion that, despite continuous improvements in the methodology for estimating dynamic models on panel data, it is not possible to unambiguously identify the best estimation method for empirical studies in corporate finance based on this type of model. However, it is possible to identify indications that, in some cases, indicate the most appropriate estimation method for the issue under consideration (*Hypothesis MH*). In conclusion, therefore, there are no grounds to reject the main hypothesis of this paper. Let us note that in the above chapter, while discussing the results of the individual simulation scenarios, the second part of the main objective of this monograph was also realised, i.e. to provide practical tips for authors of empirical articles to improve the estimation quality of the models they are considering.

CONCLUSION

This monograph considers the properties of dynamic panel model estimators in the context of corporate finance research. The primary objective of the study was to present the development of estimation methodologies for dynamic models estimated on panel data, to compare their properties in relation to corporate finance research, and to provide practical guidance for authors of empirical articles to improve the estimation quality of the models they consider. The realisation of the first part of the objective was made possible by an extensive discussion of the historical path of the development of estimation methods dedicated to dynamic panel models, an indication of their fields of use and a consideration of their diagnostic process. In addition, with reference to the literature and economic theory on the study of the optimal level of transactional corporate liquidity reserve, the validity of making comparisons of the properties of parameter estimators with a lagged dependent variable was considered.

Thanks to *Monte Carlo* simulations, which are based on panel data obtained from financial statements of listed companies in Poland posted on *NOTORIA Poland*, it was possible to verify the research hypotheses set out in the paper. The performed simulations vary the assumptions about the true size of the parameter ρ, the strength of the influence of the other explanatory variables on the dependent variable, the length of the panel adopted for the study and the distribution of the individual effect and the purely random error, examining the impact of these changes on the properties of the considered estimators.

One of the conclusions of the analysis is that the true value of the parameter with the lagged dependent variable is an important determinant of the properties of the estimators of this parameter, which is consistent with results presented in the literature. The loading on the estimators (especially Arellano-Bond) increases significantly when the true values of the parameterρ are close to unity. Furthermore, despite the low nominal loading for small values of the parameter under analysis, it can be severe and have a key impact on the conclusions of empirical studies (high relative loading with respect to the true value). In these situations, the use of estimation methods using the equation in levels (Blundell-Bond estimator and suboptimal system GMM estimator) is recommended.

The analysis also yielded the conclusion that the true magnitude of the coefficients β_k (in the sense of the model (eq. 3)) has, in principle, no significant effect on the properties of the parameter estimators with the lagged dependent variable. The exception here is when the strength of the influence of the individual regressors on the dependent variable is identical. Then, the load on the estimation methods considered based on the generalised method of moments is slightly reduced. Thus, there are no grounds for rejecting the first of the auxiliary *hypotheses* (*Hypothesis H1*) stating that the lack of variation in the strength of the influence of individual explanatory variables on the dependent variable may result in a reduction in the loadings and an improvement in the precision of estimates of the parameter standing with the lagged dependent variable. In this sense, rescaling the variables $\boldsymbol{x}_{it}$ so that the strengths of their effects on the explanatory variable are similar to each other may be a desirable procedure in understanding the properties of the estimators.

The study also identified the effect of the number of waves of the panel adopted for analysis on the properties of the estimators considered. Namely, for the shortest panels (for the analysed data set < 8), the Arellano-Bond estimators turned out to be the most adequate estimators in the sense of loading and prediction errors, but unfortunately they are characterised by a significant variance. Furthermore, for slightly longer panels ($T > 10$), the use of estimation methods using the equation in levels makes sense. In particular, the use of a suboptimal system *GMM* estimator seems to be the best solution in this case. Ultimately, Therefore, there are no grounds to reject the auxiliary hypothesis postulating that the length of the panel adopted for the study determines the choice of an adequate estimation method (*Hypothesis H2*).

Furthermore, the discussion identified a case where Blundell-Bond estimators have significantly higher relative prediction errors than Arellano-Bond estimators. This is due to the failure to meet the initial conditions assumption made for Blundell-Bond estimators (or more broadly for estimators using the equation in levels), which states that there should be no correlation between the subject's individual effect and the initial values of the explanatory variable. Otherwise, the Arellano-Bond method should be used for the final estimation of the dynamic model on panel data and estimation methods exploiting the equation in levels should not be used. Consequently, there are no grounds for rejecting the third auxiliary *hypothesis* (*Hypothesis H3*) that the presence of correlations between the individual effect of the subject and the initial values of the explanatory variable significantly narrows the spectrum of possible methods for estimating dynamic models on panel data.

The conclusions presented in the paper are unequivocal indications in favour of the main hypothesis. Finally, the indicated dependencies lead to the conclusion that there are no grounds for rejecting the main hypothesis of the study postulating that despite continuous improvements in the methodology of estimating dynamic models on panel data, the best estimation method for empirical studies in corporate finance based on this type of models cannot be unambiguously indicated. However, it is possible to identify indications that, in some cases, indicate the most appropriate estimation method for the issue under consideration.

A novel aspect of the work is the way the *Monte Carlo* simulations were carried out. They have be en based as much as possible on real data, in contrast to most works in this area, which usually make use of *an AR(p)* class model. In addition, the considerations carried out refer to the transactional liquidity reserve and use a unique dataset for the analyses, which is also a certain innovation in relation to the literature, which usually emphasises the subject of the capital structure of companies, carrying out considerations on the basis of data from the *Compustat* database. Let us also note that studies comparing the properties of estimators of dynamic panel models focus on identifying the estimation method characterised by the best properties. Meanwhile, empirical work using dynamic models estimated on panel data is so extensive that one should not point to a single most appropriate estimation method, but rather highlight narrower areas Where the superiority of a certain estimation method can indeed be highlighted. Such an approach, addressing a gap in the literature, is presented in this paper.

The results obtained can serve as a valuable source of information for researchers carrying out empirical studies in corporate finance, in particular those considering the optimal capital structure of companies, their size of transactional liquidity reserve, dividend payment policy and company investment in fixed assets. In addition, in a broader context, the paper may also be useful to authors of research in other fields who use dynamic panel models for modelling. It is also impossible to overlook a certain didactic value of *Chapter 2*, which discusses in detail the development and application of estimation methodologies dedicated to dynamic models estimated on panel data. In this sense, the work can be useful for anyone wishing to explore this topic in more depth.

The issues raised extensively discuss the problem of dynamic panel models in corporate finance research in the context of the properties of the estimators of these models. In this sense, the present work can inspire and contribute to further considerations in this area. Indeed, there are still many fields in which the content presented in the paper could be extended and enriched. Additional analysis of the properties of dynamic panel model estimators can be done with regard to the estimation of parameter sizes β_k model (eq. 3). In addition, it is important to consider in detail the properties of the suboptimal systematic estimator of the generalised method of moments for a model in which the non-delayed explanatory variables have equal power of influence on the dependent variable (in view of the higher loading noted for this method than for the Blundell-Bond estimator in the present case). A possible direction for extending the consideration is also to compare estimators dedicated to dynamic panel models with the panel conditional autoregressive ($pVAR$) model. With this approach, it would be possible to highlight the be haviour of individual variables in response to shocks to other characteristics (by means of impulse response functions).

However, carrying out the aforementioned comparison of methods would require making a number of arbitrary assumptions, due to the significantly different nature of the *pVAR* model from the standard dynamic panel model estimators.

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ANNEXES

A. TABLES

Table A.1. Basic descriptive statistics of the variable *Transactional liquidity reserve*.

Year	Average	Minimum	Maximum	Lower quartile	Median	Upper quartile	Standard deviation	Number of observations
1999	0,0676	0,0020	0,4833	0,0131	0,0284	0,0908	0,0903	119
2000	0,0563	0,0004	0,5132	0,0122	0,0262	0,0649	0,0839	127
2001	0,0386	0,0004	0,2990	0,0101	0,0246	0,0508	0,0446	124
2002	0,0474	0,0004	0,3647	0,0084	0,0216	0,0606	0,0629	156
2003	0,0477	0,0003	0,3952	0,0082	0,0230	0,0550	0,0709	199
2004	0,0608	0,0003	0,4900	0,0093	0,0240	0,0767	0,0862	218
2005	0,0768	0,0003	0,4917	0,0113	0,0335	0,1060	0,0965	221
2006	0,0803	0,0003	0,5215	0,0137	0,0412	0,1116	0,1018	257
2007	0,1027	0,0003	0,5215	0,0171	0,0462	0,1457	0,1216	273
2008	0,1068	0,0003	0,5215	0,0139	0,0427	0,1623	0,1322	306
2009	0,0896	0,0003	0,5215	0,0107	0,0373	0,1197	0,1186	335
2010	0,0855	0,0003	0,5215	0,0075	0,0383	0,1177	0,1161	378
2011	0,0821	0,0003	0,5215	0,0097	0,0338	0,1146	0,1061	456
2012	0,0770	0,0003	0,5215	0,0074	0,0298	0,0942	0,1117	519

Source: own compilation based on *NOTORIA Poland* data.

Table A.2. Summary of key characteristics, for the methods discussed in the paper for estimating dynamic models on panel data.

	The estimation method on which the approach is based	Possibility of including unobservable heterogeneity of actors in the model	Possibility of including a lagged dependent variable in the model	Possibility of including endogenous variables in the model	Possibility to take into account the restrictiveness of the dependent variable to the interval [0,1].	Assumption of no second-order correlation in first differences of purely random error	Validity of the Windmeijer adjusted variance estimator
One-step Arellano-Bond first difference estimator	*GMM*	Yes	Yes	Yes	Not	Yes	Not
Two-stage Arellano-Bond first difference estimator	*GMM*	Yes	Yes	Yes	Not	Yes	Yes
One-step systematic estimator of the generalised Blundell-Bond method of moments	*GMM*	Yes	Yes	Yes	Not	Yes	Not
Two-stage systematic estimator of the generalised Blundell-Bond method of moments	*GMM*	Yes	Yes	Yes	Not	Yes	Yes
Suboptimal system estimator of the generalized method of moments	*GMM*	Yes	Yes	Yes	Not	Yes	Not
Long-difference Instrumental Variables Estimator	*IV*	Yes	Yes	Yes	Not	Not	Not
Dynamic Panel Fractional Estimator	*ML*	Yes	Yes	Not	Yes	Not	Not
Least Square Dummy Variable Corrected Estimator	Adjusted fixed effects estimator	Yes	Yes	Not	Not	Not	Not

Source: own study.

Table A.3. Spearman correlation matrix for the continuous variables used in the study.

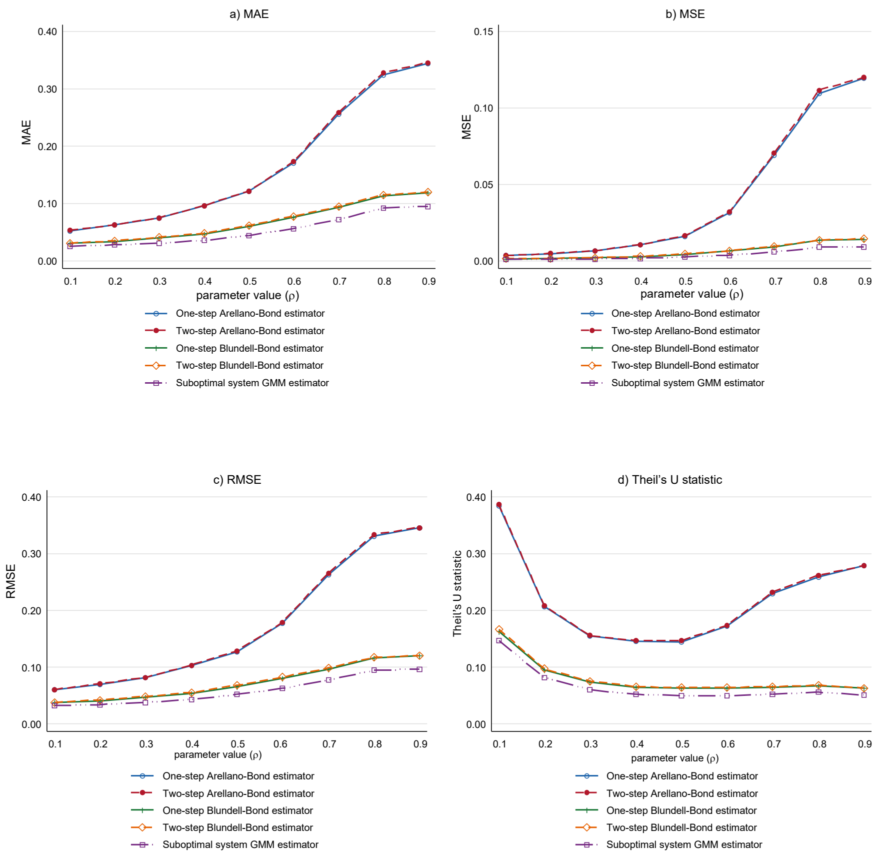
	Transaction liquidity reserve	Company size	Self-financing	Debt ratio	Funding deficit	Maturity matching	Tax rate	Net working capital	Business development opportunities	Investment expenditure	ROA
Transaction liquidity reserve	1,0000										
Company size	-0,1749*	1,0000									
Self-financing	0,1470*	0,0784*	1,0000								
Debt ratio	-0,2300*	0,0647*	-0,0257	1,0000							
Funding deficit	0,1109*	0,2092*	-0,1094*	-0,3257*	1,0000						
Maturity matching	-0,0349	0,1286*	0,0676*	-0,0524	0,0838*	1,0000					
Tax rate	0,0592*	0,1086*	0,1489*	0,0163	-0,0690*	-0,0527	1,0000				
Net working capital	0,5192*	-0,2437*	-0,0243	-0,3695*	0,0402	-0,0326	0,1077*	1,0000			
Business development opportunities	0,1464*	-0,0619*	0,0364	0,0547*	-0,1214*	-0,0755*	0,1157*	0,1118*	1,0000		
Investment expenditure	0,2157*	-0,1668*	0,0623*	-0,0291	-0,0348	-0,0895*	0,1104*	0,1977*	0,3352*	1,0000	
ROA	0,2512*	0,0219	0,3344*	-0,1950*	0,0265	-0,0416	0,2125*	0,3446*	0,2996*	0,2332*	1,0000

Correlation coefficient magnitudes that are statistically significant at the 5% significance level are marked with *. The Bonferroni correction was applied.

Source: own compilation based on *NOTORIA Poland* data.

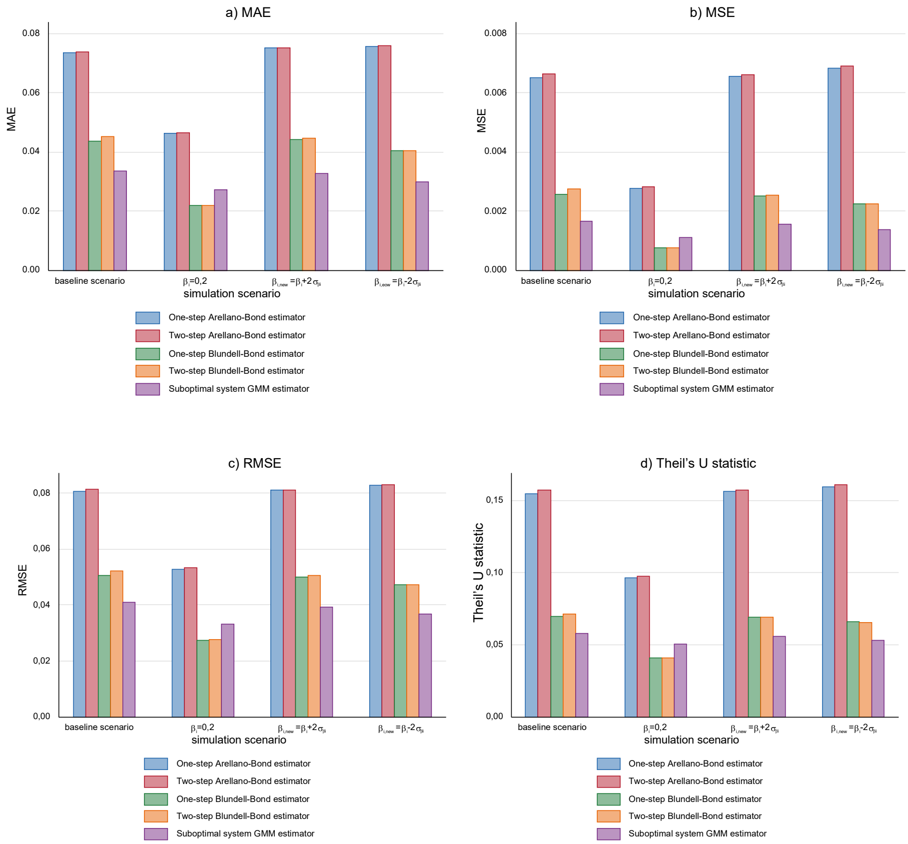
B. FIGURES

Figure B.1. Absolute measures of parameter prediction errorρ (MAE, MSE, RMSE) and Theil's U statistic for individual estimation methods depending on the true value of the parameter ρ



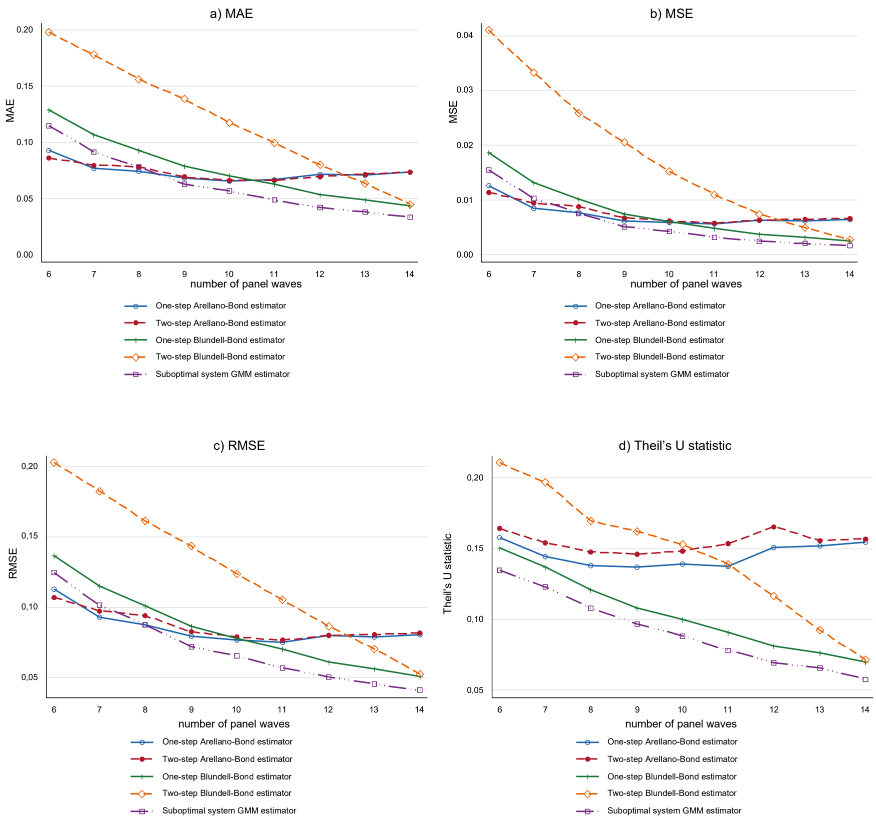
Source: own compilation based on *NOTORIA Poland* data.

Figure B.2. Absolute measures of parameter prediction error ρ (MAE, MSE, RMSE) and Theil's U statistic for the different estimation methods depending on the quantities adopted for the simulations $\beta_{k,nowy}$



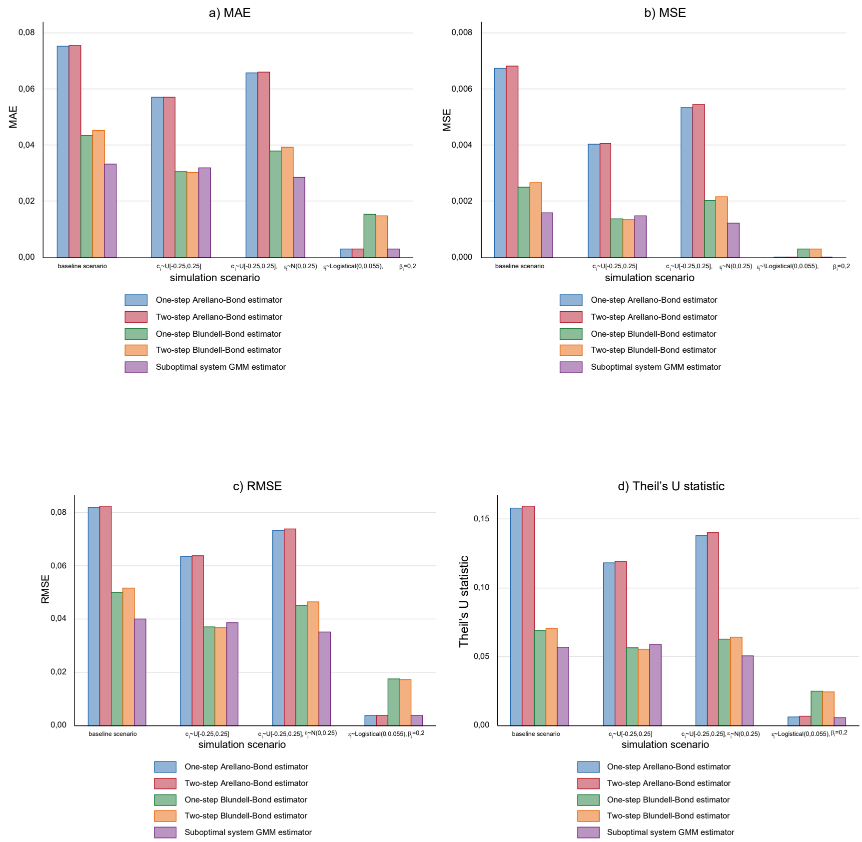
Source: own compilation based on *NOTORIA Poland* data.

Figure B.3. Absolute measures of parameter prediction error ρ (MAE, MSE, RMSE) and Theil's U statistic for individual estimation methods as a function of the number of T panel waves.



Source: own compilation based on *NOTORIA Poland* data.

Figure B.4. Absolute measures of parameter prediction error ρ (MAE, MSE, RMSE) and *Theil's U statistic* for individual estimation methods depending on the individual effect distribution and the purely random error distribution.



Source: own compilation based on *NOTORIA Poland* data.

C. PREDICTION ERROR MEASURES AND THEIL'S U STATISTIC

In order to assess the accuracy of the prediction of the parameter ρ by the considered econometric models and to compare these models with each other, the paper uses the prediction error measures most commonly used in the literature. The methods of their determination are listed below, with the formulas presented addressing the measure of the prediction error of the ρ parameter for the *Monte Carlo* simulations carried out in the study[1]. Thus, let M denote the number of iterations in the *Monte Carlo* simulations.

Prediction error measures can be broadly divided into two groups - absolute measures and relative measures. The former preserve the unit of measurement of the estimated parameter, while the latter group allows better comparison of the resulting estimates between different estimation methods.

Among the absolute measures of prediction error we distinguish:

- *Mean Absolute Error* given by the formula:[2]

$$MAE = \frac{1}{M}\sum_{i=1}^{M}|\rho - \hat{\rho}_i|, \tag{83}$$

- *Mean Square Error* given by the formula:

$$MSE = \frac{1}{M}\sum_{i=1}^{M}(\rho - \hat{\rho}_i)^2, \tag{84}$$

- *Root Mean Square Error* given by the formula:

[1] Typically, prediction error measures are presented in relation to time series forecasting, hence the formulas for their determination have a slightly different quantified notation.

[2] The true value of the parameter ρ is not indexed by the iteration number of the *Monte Carlo* simulation, as it is invariant within a given estimation method.

$$RMSE = \sqrt{\frac{1}{M}\sum_{i=1}^{M}(\rho - \hat{\rho}_i)^2}. \tag{85}$$

It should be noted that all the measures defined above do not satisfy the normality condition and that they have better properties for parameters whose values as to modulus are greater than unity. In addition, the root of the mean square error stacks up for practical reasons (less variation than *MSE*), although it does not carry any additional information value over *MSE*.

Among the relative measures of prediction error we distinguish:

- *Mean Absolute Percentage Error* given by the formula:

$$MAPE = \frac{100\%}{M}\sum_{i=1}^{M}\left|\frac{\rho - \hat{\rho}_i}{\rho}\right|, \tag{86}$$

- *Adjusted Mean Absolute Percentage Error* given by the formula:

$$AMAPE = \frac{100\%}{M}\sum_{i=1}^{M}\left|\frac{\rho - \hat{\rho}_i}{\rho + \hat{\rho}_i}\right|. \tag{87}$$

The mean relative prediction error reports the average value of prediction errors, expressed as a percentage of the actual value of the estimated parameter. *The MAPE* measure does not meet the symmetry condition, as it is higher for overestimations of predictions than for underestimations by the same absolute value. Therefore, an average adjusted relative prediction error is proposed that satisfies the symmetry condition. Both *MAPE* and *AMAPE* also satisfy the normality condition - their values belong to the interval $[0,1]$. The use of relative error measures is recommended for parameters whose values as to modulus are less than unity.

In addition, for parameters that take both positive and negative values better measures of prediction error will be absolute measures. For the parameter under consideration ρ it may be the case that the $\hat{\rho}$ estimate is negative, so absolute measures will be preferred for the study conducted. Furthermore, by assumption $\rho \leq 1$, so in summary, the most appropriate measures of prediction error for the estimates of the parameter under consideration ρ will be *MAPE* and *AMAPE*.

In this paper, in order to compare the prediction accuracy between the different estimation methods, in addition to its error measures, Theil's U statistic is used. This is an index taking values in the range $[0,2]$, given by the formula:

$$U = \frac{\sqrt{\frac{1}{M}\sum_{i=1}^{M}(\rho - \hat{\rho}_i)^2}}{\sqrt{\frac{1}{M}\sum_{i=1}^{M}\rho^2} + \sqrt{\frac{1}{M}\sum_{i=1}^{M}\hat{\rho}_i{}^2}}. \tag{88}$$

Its interpretation is as follows: when $U < 1$ ($U > 1$) is taken, the estimation method used can be considered worse (better) than if $\hat{\rho}_i = \hat{\rho}_{i-1}$ is taken for the next iteration of *the Monte Carlo* simulation. For $U = 1$ taking for the next iteration of the *Monte Carlo* simulation $\hat{\rho}_i = \hat{\rho}_{i-1}$ and using the given estimation method, are equivalent.

The monograph deals with the estimation of dynamic models on panel data, with particular emphasis on the properties of estimators used in corporate finance research. The theoretical part presents the motivation for the use of the aforementioned models and presents in detail the development of the estimation methodology of dynamic panel models, together with a comparison of their applicability and a discussion of the diagnostic process. The empirical part of the paper contains the results of Monte Carlo simulations based on real data from the financial statements of listed companies, which were taken from the Notoria database. Using these, the loading and efficiency of the estimators in question were analyzed, depending on the characteristics of the database held and the characteristics of the model under consideration. The result of the work is practical guidance for authors of empirical articles to improve the quality of the estimation of the models they are considering.

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